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The Problem of Correcting the Newtonian Mechanics

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ABSTRACT

The problem of motion of a planet or an asteroid in the gravity field of the Sun is considered with allowance for translational motion of the Solar System through the static ether. An algorithm for calculating the perihelion motion during the single orbit is found. The orbit of a planet within the framework of the classical mechanics is motionless. However, slow precession of classical planetary orbits is found in observational astronomy. The new algorithm takes into account not only the orbit geometry, but also its spatial orientation. Based on the observational data for perihelion drifts of Mercury, Earth, and Mars, the value and direction of the absolute velocity of the Sun are determined. These results are in agreement with the experimental data by Miller. Strong anomaly for the motion of the asteroid Aten is predicted. Modern techniques of practical astronomy make it possible to verify this forecast.

Keywords: Ether, Dynamics, Asteroid, Precession, Perihelion

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INTRODUCTION

According to the first Kepler law, planets move along elliptic orbits with the Sun at one of the ellipse foci. However, observational astronomy points out that the orbits undergo slow precession around the Sun toward the direction of planetary orbital motion. This phenomenon was known already to Newton who attributed it to the influence of other planets. In fact, Newton was almost right. Indeed, the orbit of Mercury, the closest planet to the Sun, drifts 5599.74 arcsec per century Brumberg (1991), and over 99% of this value is caused by factors taken into account within the framework of the classical mechanics, and only

$$(43.1 \pm 0.44)'' \tag{1}$$

amounts to the difference between the observational astronomy data and calculations based on the classical celestial mechanics algorithms. In astronomy, this residual is denoted O-C. In what follows, this quantity will be called the precession drift of simply the drift of an orbit. A number of attempts have been undertaken to explain this phenomenon. For example, Le Verrier (1811-1877) thought that there is another planet inside the Mercury orbit, which hides in the solar light. Hall (1829-1907) and Newcom (1835-1909) attempted to correct the gravitational law. As for other planets, the O-C drift is known so far with low accuracy only for Venus, Earth, and Mars. According to German astronomer H.Kienle (1895-1975), former director of Potsdam Observatory, these data are as follows (Vavilov, 1956)

$$\begin{aligned} &\text{O-C} = (-11.8 \pm 38.4)'' && \text{(for Venus)} \\ &\text{O-C} = (12.57 \pm 7.78)'' && \text{(for Earth)} \\ &\text{O-C} = (9.21 \pm 3.85)'' && \text{(for Mars)} \end{aligned} \tag{2}$$

However, American astronomer G.Clemence (1908-1974), former director of the Naval Observatory in Washington, have obtained drastically different data (Brumberg, 1991))

$$\begin{aligned} &\text{O–C} = (8.06 \pm 5.28)'' && \text{(for Venus)} \\ &\text{O–C} = (5.01 \pm 1.79)'' && \text{(for Earth)} \\ &\text{O–C} = (1.07 \pm 0.27)'' && \text{(for Mars)} \end{aligned} \tag{3}$$

The results by Kienle and Clemence have been obtained using the same, fairly inaccurate tools of practical astronomy. In addition the residual O-C per century, the angular orbital drift per one planetary orbit around the Sun is used. We denote this quantity χ and will evaluate it in radians. In 1916, Schwarzschild has obtained a rigorous solution of the Einstein equation for the motion in a centrally symmetric gravitational filed. In terms of the adopted nomenclature the Schwarzschild results is written as

$$\chi = \frac{6\pi\mu}{c^2 a(1-e^2)} \tag{4}$$

where μ $=1.328\times10^{20}$ m^3/s^2 is the product of the gravitational constant and the solar mass. Equation (4) is sometimes called the Gerber formula since P.Gerber (1854-1909) derived the same result in another way in 1898. (Gerber, 1898; Schlömilch et al., 1866). According to Gerber formula (Eq 4), the angular drift of the orbit increases with decreasing semi-major axis and predicts an increase of the angular drift of an orbit with decreasing semi-major axis a and increasing eccentricity e. Substitution of the speed of light c into Eq(4) and expression of a, the semi-major axis of the orbit, in astronomical units σ yields the following, more robust for calculations, modification of the Gerber formula:

$$\chi = \frac{0.1862}{\sigma(1-e^2)} \times 10^{-6} \tag{5}$$

Equation (5) applied to Mercury yields the following result in arcsec per century, 42.98″. This is almost ideal correspondence to the average value of experimental data (1). Expressing Kienle data (2) in radians per one orbit, we find

$\chi = (-0.352 \pm 1.15)\times10^{-6}$ (for Venus)
$\chi = (0.609 \pm 0.377)\times10^{-6}$ (for Earth) (6)
$\chi = (0.840 \pm 0.351)\times10^{-6}$ (for Mars)

The corresponding evaluation of Clemence data (3) results in

$\chi = (0.240 \pm 0.158)\times10^{-6}$ (for Venus)
$\chi = (0.243 \pm 0.087)\times10^{-6}$ (for Earth) (7)
$\chi = (0.0976 \pm 0.0246)\times10^{-6}$ (for Mars)

Note that neither Kienle nor Clemence cannot be considered close to actual ones since these data have been obtained before the époque of using radar techniques in observational astronomy for determining current coordinates of celestial bodies. Radars have been started to be used in observational astronomy since the sixties of the last century. This revolutionized the observational astronomy since the errors of observational data on celestial-body motion reduced by two orders of magnitude.

At the same time, no refined observational data on O-C or χ for Venus, Earth, and Mars were published. Along with the development of radar techniques, correction coefficients (Pitjeva, 2005) absent in the Einstein general relativity started to be introduced.

As a result, a new algorithm for calculating the celestial body motion has been developed. This algorithm was called post-Newtonian (Pitjeva, 2005) was laid in the basis for finding ephemeris. Of course, any algorithms correcting previous ones should be welcomed. At the same time, some absolutely unforeseen problems appeared. These problems are such that one fails to formulate them strictly.

EFFECT DUE TO NON-SHPERICITY OF THE SUN

It has been found long ago that centrifugal forces tend to shrink the Sun along its rotational axis. However, only (Dicke, 1970) was the first to guess right that the solar non-sphericity should affect the precession drift of planetary orbits. However, two following factors complicate the problem. First, density distribution over the solar volume is not known. Second, the Sun rotates differentially, i.e., its inner parts have higher angular velocity than equatorial ones. The angular-velocity profile over the Sun is not known, as well. Shapiro (1965, 1968, 1971) tried to determine the quadrupole moment of the Sun using the motion of the asteroid Icarus. At first, it was assumed that the actual value of χ for Icarus can exceed the general-relativistic forecast by 50% (Dicke, 1970). The large paper (Shapiro, Smith, Ash, & Herrick, 1971) contains 9 journal pages with data tables on the Icarus motion. However, papers (Shapiro, 1965; Shapiro, Ash, & Smith, 1968; Shapiro et al., 1971) do not mention any specific information on the values of O-C or χ for this asteroid. Shapiro (1965) gives the formula describing the “quadrupole” drift of an orbit per terrestrial year. It can be transformed to find the drift χ_2 per one orbit around the Sun. In terms of our notations, the new formula looks as follows:

$$\chi_2 = J_2^* E \times 10^{-6} \tag{8}$$

where E is the dimensionless function of the orbital and solar parameters,

$$E = \frac{0,1}{\sigma^2(1-e^2)^2}\{5\cos^2 i\cos^2 i_S - 1 + (3-5\cos^2 i)\cos^2(\Omega - \Omega_S)\sin^2 i_S + (4-5\cos^2 i)\cos(\Omega - \Omega_S)\sin 2i_S \,\mathrm{ctn}\, i\} \tag{9}$$

Here, J_2^* is the dimensionless quadrupole moment of the Sun, i_S is the angle between the solar-equator plane and the ecliptic plane, Ω_S is ascending node - perihelion angle of the Sun

$$i_S = 7.25°;\ \Omega_S = 75.06°$$

i is the inclination of the orbit with respect to the ecliptic plane, and Ω is the ascending node-perihelion angle of the celestial-body orbit. Equation (9) is inapplicable to the Earth ($i = 0$, $\mathrm{ctn}\, i = \infty$, and the parameter Ω is undetermined). This formula should be transformed with allowance for

$$\cos(\Omega - \Omega_S) = 0\ ,\quad \cos(\Omega - \Omega_S)\,\mathrm{ctn}\, i = 1$$

As a result, Eq (9) is reduced to the following formula applicable to the Earth,

$$E = \frac{0,1}{\sigma^2(1-e^2)^2}(5\cos^2 i_S - 1 - \sin 2i_S) \tag{10}$$

Dicke estimated the quadrupole drift χ_2 for Mercury to be 9% of the total drift χ (Dicke, 1970). Since the average valued O-C (1) for Mercury corresponds to

$$\chi = 0.503 \times 10^{-6} \qquad (11)$$

9% of this quantity is

$$\chi_2 = 0.0453 \times 10^{-6} \qquad (12)$$

Now Eq (9) can be used to find the value of E for Mercury and then, based on Eq (8), calculate the corresponding value of the solar quadrupole moment J_2^*

$$E = 1.558; \quad J_2^* = 0.0291 \qquad (13)$$

The accuracy of the obtained value of J_2^* can be justifiably doubted. However, the alternative is either to completely ignore the fact of the solar oblateness or accept the Dicke estimate. Table 1 lists the values of E and χ_2 for four planets and six asteroids if J_2^* is given by Eq (10). Here, the orbital parameters σ, e, Ω, ω, and i are pointed out, as well. The angle ω is called ascending node - perihelion angle. This parameter will be used below. It is seen that the nonsphericity of the Sun stipulates increases in the precession drifts of the orbits of Phaeton and Icarus most of all. The drift χ_2 for Venus and Mars is small and has the opposite sign. Figures 1 and 2 clearly illustrate the geometry of the orbits of all ten celestial bodies and their spatial locations.

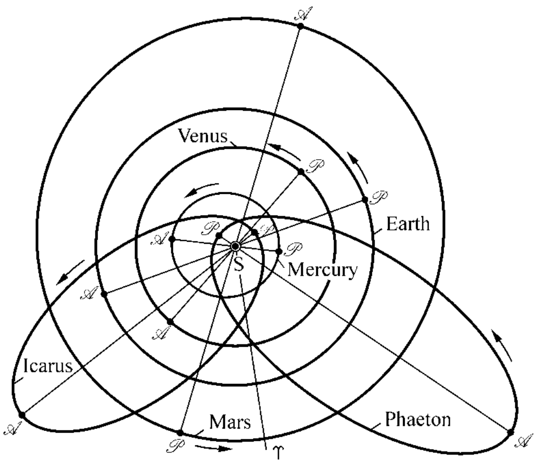

Fig 1. Orbits of four planets and two asteroids projected onto the ecliptic plane

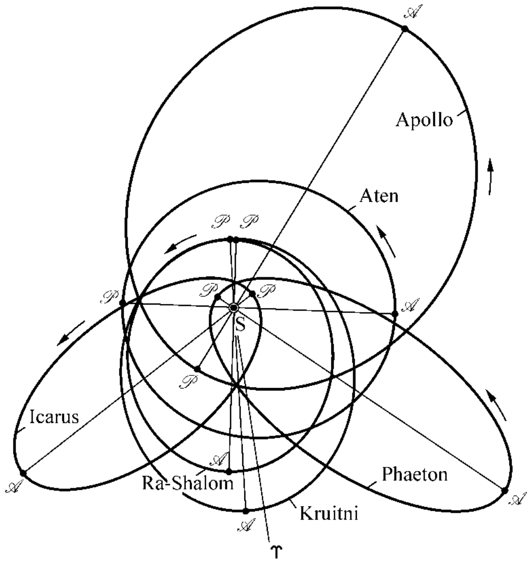

Fig 2. Orbits of six asteroids projected onto the ecliptic plane

It is seen that the celestial bodies selected for Table 1 vary strongly in size, eccentricity and position of the orbit relative to the spring equinox point ϒ. The symbols 𝒫 and 𝒜 denote the perihelion and aphelion, respectively. The major axis of the orbit connecting perihelion and aphelion is called the apse. Ten selected celestial bodies are also strongly different in their inclinations i to the ecliptic (Table 1).

Table 1. The orbital parameters σ, e, Ω, ω, i and values of E and χ_2 for planets and asteroids

Object	σ	e	Ω(°)	ω(°)	i(°)	E	$\chi_2 \cdot 10^6$
Mercury	0.3871	0.2056	48.033	29.033	7	1.558	0.0453
Venus	0.7233	0.00677	76.455	54.764	3.394	-0.0526	-0.0015
Earth	1	0.0167	Ω + ω = 102.51		0	0.367	0.0107
Mars	1.5237	0.0934	49.365	286.234	1.85	-0.134	-0.0039
Icarus	1.0779	0.8268	88.079	31.296	22.85	2.59	0.0754
Phaeton	1.271	0.890	265.4	322.0	22.2	4.83	0.141
Ra-Shalom	0.832	0.437	170.92	335.98	15.75	0.798	0.0232
Aten	0.967	0.183	108.64	147.95	18.33	0.355	0.0103
Kruitni	0.998	0.519	126.28	43.74	19.81	0.592	0.0172
Apollo	1.471	0.560	35.9	285.67	6.35	0.216	0.0063

DIFFERENTIAL EQUATIONS OF MOTIONS OF PLANETS

Figure 3 shows an elliptical orbit $\mathscr{L}$ of a planet or an asteroid. The translational velocity $\mathbf{v}_0$ of the Sun, whose magnitude and direction are to be determined, makes the angle ϑ with the orbital plane.

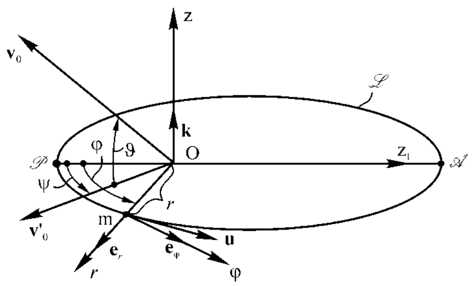

Fig 3. Planetary orbit and the vector $\mathbf{v}_0$ of the translation velocity of the Sun

The angle ψ determines the position of the vector $\mathbf{v'}_0$, i.e., the projection of the velocity $\mathbf{v}_0$ onto the orbital plane. The axis z with the ort $\mathbf{k}$ is orthogonal to the orbital plane; $\mathbf{e}_r$ and $\mathbf{e}_\varphi$ are the orts of the polar coordinates r and φ, respectively; and $\mathbf{u}$ is the orbital velocity of the body. It is seen that

$$\mathbf{v}_0 = (\mathbf{e}_r\sigma_2 - \mathbf{e}_\varphi\sigma_1 + \mathbf{k}\sin\vartheta), \quad \mathbf{u} = \mathbf{e}_r\dot{r} + \mathbf{e}_\varphi r\dot{\varphi},$$
$$\mathbf{v} = \mathbf{v}_0 + \mathbf{u} = \mathbf{e}_r(\dot{r} + v_0\sigma_2) + \mathbf{e}_\varphi(r\dot{\varphi} - v_0\sigma_1) + \mathbf{k}v_0\sin\vartheta,$$
$$\sigma_1 = \cos\vartheta\sin(\varphi - \psi), \quad \sigma_2 = \cos\vartheta\sin(\varphi - \psi) \tag{14}$$

where the dot above the letter denotes a derivative with respect to the time t. In the equation of relativistic dynamics,

$$\frac{d}{dt}\left(\frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}}\right) = \mathbf{F} \tag{15}$$

and $\mathbf{v}$ denotes the relative velocity of the object. However, according to Eq (14), $\mathbf{v}$ is the absolute velocity of the point in our case, i.e. Eq (15) is written in the ether frame. It is assumed that v_0 is an order of magnitude greater that u. In the case of high-velocity charged particles for which u is two orders of magnitude greater than v_0, the absolute velocity $\mathbf{v}$ is close to the relative velocity $\mathbf{u}$, so that Eq (15) reduces to the relativistic one. Hence, our interpretation of Eq (15) does not contradict particle-accelerator experiments. Equation (15) is the only new element in the classical-mechanics technique. Calculation of the derivative in Eq (15) yields

$$\frac{1}{\sqrt{1-v^2/c^2}}\left\{\frac{d\mathbf{v}}{dt} + \frac{\mathbf{v}}{c^2(1-v^2/c^2)}\left(\mathbf{v}\cdot\frac{d\mathbf{v}}{dt}\right)\right\} = \frac{\mathbf{F}}{m}$$

The second term in the parentheses is of the order of v^2/c^2, so the expression $(1 - v^2/c^2)$ can be approximated by unity. In addition, it is seen from Eq (14) for $\mathbf{v}$ that

$$\frac{d\boldsymbol{v}}{dt} = \frac{d\boldsymbol{u}}{dt}$$

and the considered equation becomes simpler:

$$\frac{1}{\sqrt{1-v^2/c^2}}\left\{\frac{d\mathbf{u}}{dt} + \frac{\mathbf{v}}{c^2}\left(\mathbf{v}\cdot\frac{d\mathbf{u}}{dt}\right)\right\} = \frac{\mathbf{F}}{m} \tag{16}$$

With allowance for Eq (14) we find

$$\frac{d\mathbf{u}}{dt} = \mathbf{e}_r(\ddot{r} - r\dot{\varphi}^2) + \frac{\mathbf{e}_\varphi}{r}\frac{d}{dt}(r^2\dot{\varphi}),$$
$$\left(\mathbf{v}\cdot\frac{d\mathbf{u}}{dt}\right) = v_0\left\{\left(\frac{\dot{r}}{v_0} + \sigma_2\right)(\ddot{r} - r\dot{\varphi}^2) + \left(\frac{r\dot{\varphi}}{v_0} - \sigma_1\right)\frac{1}{r}\frac{d}{dt}(r^2\dot{\varphi})\right\}$$

Let us introduce the small parameter ε:

$$\varepsilon = \frac{v_0^2}{2c^2} \tag{17}$$

and verify the validity of the equalities

$$\frac{v^2}{c^2} = 2\varepsilon\left\{\left(\frac{\dot{r}}{v_0} + \sigma_2\right)^2 + \left(\frac{r\dot{\varphi}}{v_0} - \sigma_1\right)^2 + \sin^2\vartheta\right\},$$
$$\frac{1}{\sqrt{1-v^2/c^2}} \approx 1 + \frac{v^2}{2c^2} =$$
$$= 1 + \varepsilon\left[\sin^2\vartheta + \left(\frac{\dot{r}}{v_0} + \sigma_2\right)^2 + \left(\frac{r\dot{\varphi}}{v_0} - \sigma_1\right)^2\right]$$

Since

$$\frac{\mathbf{F}}{m} = -\frac{\mu}{r^2}\mathbf{e}_r$$

where μ has the same meaning as in Gerber formula (4), the vector equation (16) with allowance for the obtained formulas is firstly reduced and then yields three scalar equations

$$\left\{1 + \varepsilon\left[\sin^2\vartheta + 3\left(\sigma_2 + \frac{\dot{r}}{v_0}\right)^2 + \left(\sigma_1 - \frac{r\dot{\varphi}}{v_0}\right)^2\right]\right\}(\ddot{r} - r\dot{\varphi}^2) -$$
$$-2\varepsilon\left(\sigma_2 + \frac{\dot{r}}{v_0}\right)\left(\sigma_1 - \frac{r\dot{\varphi}}{v_0}\right)\frac{1}{r}\frac{d}{dt}(r^2\dot{\varphi}) = -\frac{\mu}{r^2},$$
$$\left\{1 + \varepsilon\left[\sin^2\vartheta + \left(\sigma_2 + \frac{\dot{r}}{v_0}\right)^2 + 3\left(\sigma_1 - \frac{r\dot{\varphi}}{v_0}\right)^2\right]\right\}\frac{1}{r}\frac{d}{dt}(r^2\dot{\varphi}) -$$
$$-2\varepsilon\left(\sigma_1 - \frac{r\dot{\varphi}}{v_0}\right)\left(\sigma_2 + \frac{\dot{r}}{v_0}\right)(\ddot{r} - r\dot{\varphi}^2) = 0 \tag{18}$$
$$z = 2\varepsilon r\left(\frac{\dot{r}}{v_0} + \sigma_2\right)\sin\vartheta$$

The last equation makes it possible to calculate the deviation z of a planet from the classical-orbit plane. Concerning the problem of determination of the precession drift of the orbit, only the first and second equations are to be used. Compare these equations with the classical-mechanics equations

$$\ddot{r}_0 - r_0\dot{\varphi}_0^2 = -\frac{\mu}{r_0^2}, \quad r_0^2\dot{\varphi}_0 = \lambda, \tag{19}$$

where

$$\lambda = \sqrt{\mu\, a\,(1 - e^2)}. \tag{20}$$

Now we replace the variables r and φ by the variables r_1 and φ_1 according to the formulas

$$r = r_0 + \varepsilon\, r_1, \varphi = \varphi_0 + \varepsilon\varphi_1, \tag{21}$$

where r_0 and φ_0 are the solutions of Eq(19) corresponding to the motion along the elliptic orbit

$$r_0 = \frac{a\,(1-e^2)}{1+e\cos\varphi_0}. \tag{22}$$

The angle φ_0 is reckoned from the perihelion toward the direction of the planet motion. In what follows, the terms of the order of $\varepsilon^2$are always neglected. Using the second equation of Eq (19) and Eqs (20) and (22), we check the validity of the relationships

$$\begin{gathered}\dot r_0 = \dot\varphi_0 \frac{dr}{d\varphi_0} = \frac{\lambda e\sin\varphi_0}{a(1-e^2)}, \quad \frac{\dot r_0}{v_0} = \frac{\lambda e\sin\varphi_0}{v_0 a(1-e^2)} = \delta\, e\sin\varphi_0,\\ \frac{r_0\dot\varphi_0}{v_0} = \frac{\lambda(1+e\cos\varphi_0)}{v_0 a(1-e^2)} = \delta(1+e\cos\varphi_0),\\ \delta = \frac{\lambda}{v_0 a(1-e^2)} = \frac{1}{v_0}\sqrt{\frac{\mu}{a(1-e^2)}} = \frac{29{,}79}{v_0\sqrt{\sigma\nu}}, \quad \nu = 1-e^2.\end{gathered} \tag{23}$$

Here, δ and ν are dimensionless parameters, while the resulting expression for δ implies using the translational velocity v_0 in km/s. Now denoting

$$g_1 = e\sin\varphi_0, \quad g_2 = 1 + e\cos\varphi_0, \tag{24}$$

we arrive from Eq (18) at

$$\begin{cases}\{1+\varepsilon[\sin^2\vartheta + 3(\sigma_2+\delta g_1)^2 + (\sigma_1-\delta g_2)^2]\}(\ddot r - r\dot\varphi^2)\\ \quad -2\varepsilon(\sigma_2+\delta g_1)^2(\sigma_1-\delta g_2)\dfrac{1}{r}\dfrac{d}{dt}(r^2\dot\varphi) = -\dfrac{\mu}{r^2},\\ \{1+\varepsilon[\sin^2\vartheta + (\sigma_2+\delta g_1)^2 + 3(\sigma_1-\delta g_2)^2]\}\dfrac{1}{r}\dfrac{d}{dt}(r^2\dot\varphi)\\ \quad -2\varepsilon(\sigma_1-\delta g_1)(\sigma_2+\delta g_2)(\ddot r - r\dot\varphi^2) = 0.\end{cases} \tag{25}$$

The approximate equalities

$$\begin{gathered}\ddot r - r\dot\varphi^2 = \ddot r_0 + \varepsilon\ddot r_1 - (r_0+\varepsilon r_1)(\dot\varphi_0+\varepsilon\dot\varphi_1)^2 \approx\\ \approx \ddot r_0 - r_0\dot\varphi_0^2 + \varepsilon(\ddot r_1 - 2r_0\dot\varphi_0\dot\varphi_1 - \dot\varphi_0^2 r_1),\\ \frac{\mu}{r^2} = \frac{\mu}{(r_0+\varepsilon r_1)^2} \approx \frac{\mu}{r_0^2} - \varepsilon\frac{2\mu}{r_0^3}r_1, \quad \frac{1}{r} = \frac{1}{r_0+\varepsilon r_1} \approx \frac{1}{r_0} - \varepsilon\frac{r_1}{r_0^2},\\ \frac{1}{r}\frac{d}{dt}(r^2\dot\varphi) = \frac{1}{r_0+\varepsilon r_1}\frac{d}{dt}[(r_0+\varepsilon r_1)^2(\dot\varphi_0+\varepsilon\dot\varphi_1)] \approx\\ \approx \frac{1}{r_0}\frac{d}{dt}(r_0^2\dot\varphi_0) + \frac{\varepsilon}{r_0}\frac{d}{dt}(r_0^2\dot\varphi_1 + 2r_0\dot\varphi_0 r_1)\end{gathered}$$

allow one to replace system of equations (25) with approximate equations (19) and two equations in terms of the new variables r_1 and φ_1

$$\begin{gathered}\ddot r_1 - 2r_0\dot\varphi_0 - \left(\dot\varphi_0^2 + \frac{2\mu}{r_0^3}\right)r_1 = \frac{\mu}{r_0^2}\theta_1,\\ \frac{d}{dt}(r_0^2\dot\varphi_1 + 2r_0\dot\varphi_0 r_1) = \frac{\mu}{r_0}\theta_2\end{gathered} \tag{26}$$

Here, θ_1 and θ_2 are the functions of the polar angle φ_0

$$\begin{gathered}\theta_1 = \sin^2\vartheta + 3\,(\sigma_2+\delta g_1)^2 + (\sigma_1-\delta g_2)^2\\ \theta_2 = -2\,(\sigma_1-\delta g_2)\,(\sigma_2+\delta g_1)\end{gathered} \tag{27}$$

Since the desired quantity is the deviation of a planet from its classical elliptic orbit, the argument t can be expressed in terms of the variable φ_0. The operator relation

$$\frac{d}{dt} = \dot\varphi_0\frac{d}{d\varphi_0} = \frac{\lambda}{r_0^2}\frac{d}{d\varphi_0}$$

and the second equality in Eq (19) allow for making the transformations

$$\begin{gathered}\ddot r_1 = \frac{d}{dt}\left(\dot\varphi_0\frac{dr_1}{d\varphi_0}\right) = \dot\varphi_0\frac{dr_1}{d\varphi_0} + \dot\varphi_0^2\frac{d^2r_1}{d\varphi_0^2}, \dot\varphi_0^2 = \frac{\lambda^2}{r_0^4},\\ \ddot\varphi_0 = -\frac{2\lambda^2}{r_0^5}\frac{dr_0}{d\varphi_0}, \ddot r_1 = \frac{\lambda^2}{r_0^4}\cdot\frac{d^2r_1}{d\varphi_0^2} - \frac{2\lambda^2}{r_0^5}\cdot\frac{dr_0}{d\varphi_0}\cdot\frac{dr_1}{d\varphi_0},\\ \dot\varphi_1 = \frac{\lambda}{r_0^2}\frac{d\varphi_1}{d\varphi_0}, 2\frac{\lambda}{r_0}\dot\varphi_1 = \frac{2\lambda^2}{r_0^3}\cdot\frac{d\varphi_1}{d\varphi_0}, r_0\dot\varphi_1 = \lambda\frac{d\varphi_1}{d\varphi_0},\\ \frac{d}{dt}\left(r_0^2\dot\varphi_1 + 2\frac{\lambda}{r_0}r_1\right) = \frac{\lambda^2}{r_0^2}\left(\frac{d^2\varphi_1}{d\varphi_0^2} + \frac{2}{r_0}\frac{dr_1}{d\varphi_0} - \frac{2}{r_0^2}\frac{dr_0}{d\varphi_0}r_1\right)\end{gathered}$$

These formulas make it possible to reduce Eqs (26) to the form

$$\begin{gathered}\frac{d^2r_1}{d\varphi_0^2} - \frac{2}{r_0}\cdot\frac{dr_0}{d\varphi_0}\cdot\frac{dr_1}{d\varphi_0} - 2r_0\Phi - \left(1 + \frac{2\mu\, r_0}{\lambda^2}\right)r_1 = \frac{\mu\, r_0^2}{\lambda^2}\theta_1,\\ \frac{d\Phi}{d\varphi_0} + \frac{2}{r_0}\frac{dr_1}{d\varphi_0} - \frac{2}{r_0^2}\frac{dr_0}{d\varphi_0}r_1 = \frac{\mu r_0}{\lambda^2}\theta_2,\end{gathered} \tag{28}$$

where $\Phi = d\varphi_1/d\varphi_1$ Let us introduce the dimensionless variables ρ_0 and ρ_1 instead of r_0 and r_1: $r_0 = a\rho_0$, $r_1 = a\rho_1$. Here, as before, a is the semi-major axis of the classical orbit. Based on Eqs (22) and (23), we find

$$\rho_0 = \frac{\nu}{1+e\cos\varphi_0}, \quad \frac{d\rho_0}{d\varphi_0} = \rho_0^2\frac{e}{\nu}\sin\varphi_0$$

In terms of dimensionless functions

$$\begin{gathered}P = \frac{\mu a}{\lambda^2}\rho_0 = \frac{1}{g_2}, g = \frac{e\sin\varphi_0}{\nu} = \frac{g_1}{\nu},\\ \theta = g_1 P, \Lambda = 1 + \frac{2}{1+e\cos\varphi_0} = 1 + 2P\end{gathered} \tag{29}$$

the system of equations (28) reduces to its final form

$$\begin{gathered}\frac{d^2\rho_1}{d\varphi_0^2} - 2\theta\frac{d\rho_1}{d\varphi_0} - \Lambda\rho_1 - 2\rho_0\Phi = \rho_0 P\theta_1,\\ \frac{d\Phi}{d\varphi_0} + \frac{2}{\rho_0}\frac{d\rho_1}{d\varphi_0} - 2g\rho_1 = P\theta_2\end{gathered} \tag{30}$$

ALGORITHM FOR CALCULATING THE DRIFT χ_1

The direction of the translational-velocity vector $\mathbf{v}_0$ (Fig. 4) in the ecliptic coordinates is determined by the ecliptic latitude β, i.e., the angle between the ecliptic plane and the vector $\mathbf{v}_0$, and the ecliptic longitude λ (this symbol in the previous meaning given by Eq (20) is not needed anymore).

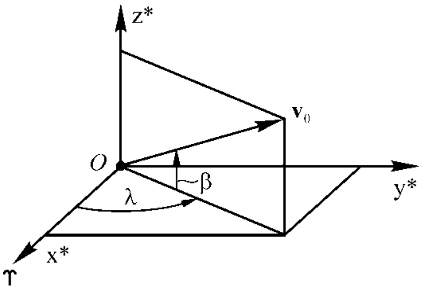

Fig 4. Direction of the vector $\mathbf{v}_0$ in ecliptic coordinates

The ecliptic longitude is the angle λ between the direction to the spring-equinox point ϒ and the projection of the vector $\mathbf{v}_0$ onto the ecliptic plane. Let us determine the spatial orientation of the orbit (Fig 5). The points of intersection of the orbit with the ecliptic plane are called the ascending and descending nodes.

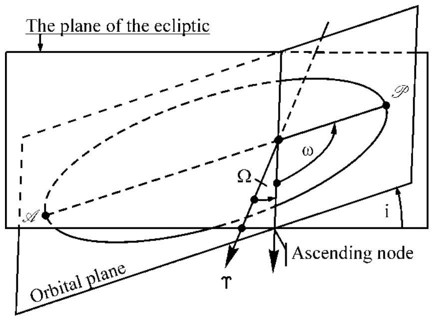

Fig 5. Specifying the spatial orientation of a planetary orbit

By passing the ascending angle, a planet is shifted toward the north. The angle i between the orbital and ecliptic planes is called the inclination. All planets and asteroids rotating in the same direction as the Earth have $i > 0$. The angle Ω between the direction of the spring-equinox point ϒ and the ascending node is called the ascending-node longitude. The angle ω between the direction to the ascending angle and perihelion is called the perihelion argument.

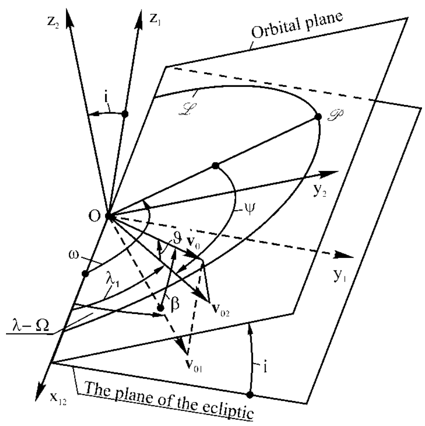

Fig. 6. Determination of the angles ϑ and ψ based on the angles Ω, ω, i, β, and λ

The functions θ_1 and θ_2 given by Eq (27) and entering the right-hand sides of Eq(30) have the meaning of dimensionless forces perturbing the motion of a planet along the classical elliptic orbit. These functions include σ_1 and σ_2 (14) in which the angles ϑ and ψ are contained (Fig 6).

Therefore, it is necessary to construct an algorithm for calculating σ_1 and σ_2 given five angles β, λ, Ω, ω, and i. Figure 6, in which the angle ψ is negative, makes it possible to solve this problems using elementary geometric considerations. In what follows, only final results for the complete algorithm for calculating the precession drift χ_1 per one orbital cycle.

$$\nu = 1 - e^2, \delta = \frac{29.79}{v_0\sqrt{\sigma\nu}}, \quad \varphi_0 = \lambda - \Omega,$$
$$\alpha_1 = \sin\varphi_0\cos\beta\cos i + \sin\beta\sin i,$$
$$\alpha_2 = \cos\varphi_0\cos\beta,$$
$$\alpha = \sqrt{\alpha_1^2 + \alpha_2^2},$$
$$\zeta_1 = \alpha_1/\alpha,$$
$$\zeta_2 = \alpha_2/\alpha, \ \ \xi_1 = \zeta_1\cos\omega - \zeta_2\sin\omega, \ \ \xi_2 = \zeta_2\cos\omega + \zeta_1\sin\omega,$$
$$g_1 = e\sin\varphi_0, \ \ g_2 = 1 + e\cos\varphi_0, \ \ g = g_1/\nu, \ \ P = 1/g_2,$$
$$\rho_0 = \nu P, \quad \theta = g_1 P, \ \ \Lambda = 1 + 2P,$$
$$\sigma_1 = \alpha(\xi_2\sin\varphi_0 - \xi_1\cos\varphi_0), \sigma_2 = \alpha(\xi_2\cos\varphi_0 + \xi_1\sin\varphi_0)$$

The symbolsα, ζ_1, ζ_2, ξ_1, and ξ_2 have simple trigonometric meaning,

$$\alpha = \cos\vartheta, \quad \zeta_1 = \sin(\psi + \omega), \quad \zeta_2 = \cos(\psi + \omega),$$
$$\xi_1 = \sin\psi, \quad \xi_2 = \cos\psi \tag{31}$$

Upon performing the above operations, the functions θ_1 and θ_2 given by Eq (27) are programmed and the equations given by Eq (30) are composed. System of equations (30) was integrated using the Runge-Kutta method in the interval from $\varphi_0 = 0$ to $\varphi_0 = 2\pi$. The marching method turned out to be numerically unstable. The boundary conditions

$$\frac{d\rho_1}{d\varphi_0} = 0 \text{ for } \varphi_0 = 0, \qquad \frac{d\rho_1}{d\varphi} = 0 \text{ for } \varphi_0 = 2\pi \tag{32}$$

are evident since the calculations are started and terminated at the perihelion. The initial values of the functions ρ_1 and Φ for $\varphi_0 = 0$ were found as a consequence of fulfillment of the second condition given by Eq (32). The interval from 0 to 2π was divided into 400 equal steps. The derivative

$$\frac{d\rho_1}{d\varphi_0}(\varphi = 2\pi) = f[\rho_1(0), \Phi(0)]$$

Was nullified by the gradient method until the following criterion was achieved

$$\frac{d\rho_1}{d\varphi_0}(\varphi = 2\pi) < 10^{-10}$$

It was found that the functions $\rho_1(\varphi_0)$ and $\Phi(\varphi_0)$ are not strictly periodic, which results in stable discrepancies for each celestial body

$$\Delta\rho_1 = \rho_1(2\pi) - \rho_1(0), \Delta\Phi = \Phi(2\pi) - \Phi(0) \tag{33}$$

This effect is caused by the dependence of the left-hand side of Eq (15) on the ratio v^2/c^2. Indeed, consider a classical orbit with such an orientation that the velocity $\mathbf{v}_0$ is projected on the orbital plane strictly along the apse. According to the classical mechanics, the values of velocities and accelerations are symmetric with respect to the apse. However, according to our theory, that ratio v^2/c^2 is different at both sides of the apse. This results in an asymmetric distortion of the classical elliptic orbit and violation of the symmetry of the velocity and acceleration with respect to the apse. This stipulates such a deformation of the classical zero-eccentricity orbit that, in contrast to a pure circular orbit, its precession becomes meaningful. However, discrepancies (33) in terms of distances and velocities have actually unobservable values. The drift χ_1 was calculated in two steps:

$$\varepsilon = \frac{1}{2}\left(\frac{v_0}{300}\right)^2 \times 10^{-6}, \quad \chi_1 = \varepsilon \int_0^{2\pi} \Phi(\varphi)\, d\varphi \qquad (34)$$

where the integral was evaluated using the Simpson formula. The translational velocity v_0 in km/s should be substituted in the formulas for both ε and δ.

RESULTS AND CONCLUSIONS

Three different sources indicative of the translational motion of the Solar System are known. We should check in which case the motion relative to the ether is taken into account.

- Motion relative to the cosmic microwave background with the parameters

$$v_0 = 366 \text{ km/s}, \quad \beta = -11.25°; \quad \lambda = 171.8° \qquad (35)$$

- Data by Prof. E.I. Shtyrkov (2005) obtained by observing aberration of electromagnetic waves from a geostationary satellite

$$v_0 = 600 \text{ km/s}, \quad \beta = 67.07°; \quad \lambda = 90.01° \qquad (36)$$

- Data by Dayton Miller (1933) obtained using the Michelson interferometer. When operated in the atmospheric air, this device makes it possible to point out two diametrically opposite infinitely distant points on the sky. The translational velocity $\mathbf{v}_0$ is directed along the line connecting these points.

However, the Michelson interferometer cannot determine one of these two points toward which the velocity $\mathbf{v}_0$ is directed. Miller selected the southern point without clear explanations. However, the calculations showed that the velocity $\mathbf{v}_0$ is directed to the northern Miller point located near the northern ecliptic pole (Miller, 1933)

$$\beta = 82.81°, \quad \lambda = 139.07° \qquad (37)$$

The effect discovered by Miller was confirmed by the Michelson, Pease, and Pearson (1929). Based on the weak dependence of the observed effect on the direction of the Earth orbital velocity, Miller indirectly estimated the translational velocity v_0 to amount to 208 km/s. Coordinates (37) of the Miller point were determined also fairly approximately based on statistical analysis of experimental data exposed to strong interference. Calculations using parameters (35) showed that the drift χ_1 for Mercury was over valuated almost fourfold (1.88×10^{-6}), the corresponding values for Venus and Earth amount to 42×10^{-6} and 20×10^{-6}, and $\chi_1 = 0.047\times10^{-6}$ for Mars. These results contradict to the data by both Kienle (6) and Clemens (7). This means that the frame of the cosmic microwave background has no relation to the ether. The data by Shtyrkov (36) yielded the drift χ_1 for Mercury of 1.48×10^{-6}, which is thrice as large as experimental value (11). The drifts for Earth and Venus turned out to be reverse and amount to -24.2×10^{-6} and -2.69×10^{-6}, respectively. As for Mars, $\chi_1 = 1.69\times10^{-6}$ is too large even for Kienle data (6). Data (36) are indicative of the need in correcting either registration or processing of the observational data, although the Shtyrkov method for evaluation of v_0, β, and λ seems to be the most reliable at present.

The best results were obtained by the Miller point (37). For v_0 = 208 km/s we found $\chi_1 = 0.439\times10^{-6}$ for Mercury, which is about 13% less than the actual value given by Eq (11). These 13% could be attributed to the quadrupole moment of the Sun. The Earth drift ($\chi_1 = 0.998\times10^{-6}$) does not contradict Kienle data (6), whereas the Martian drift ($\chi_1 = 0.112\times10^{-6}$) is in agreement even with Clemens data (7). However, the Venus drift ($\chi_1 = 1.96\times10^{-6}$) falls even out the range of Eq (6) by a factor of 2.5. Meanwhile, it is undoubtedly obvious that the coordinates of the point to which the translational velocity of the Sun is directed have been determined by Miller with surprisingly small error bearing in mind that such a device as the Michelson interferometer is strongly sensitive to interference. We decided to search for more perfect combination of the parameters v_0, β, and λ beginning with Miller point (37) and performing two following steps. Firstly the angles β and λ were varied for the fixed velocity v_0 = 208 km/s to achieve the proper proportion between the drifts of Venus, Earth, and Mars. Then the values of v_0 was found using the condition

$$\chi_1 = 0.458\times10^{-6} \quad \text{(for Mercury)} \qquad (38)$$

This was done by summing χ_1(38) and χ_2 (12) and ensuring that χ corresponding to Eq (11) is reached. Calculations showed that, as a rule, deviations from the northern ecliptic ($\beta = 90°$) in comparison with Miller point (37) do not improve, but worsen the results. Thus, points inside such a vicinity of the northern ecliptic pole whose boundary is determined by Miller point (37) ($\beta \approx 83°$) were mainly tested. As for the ecliptic longitude λ, the equatorial coordinates used by (Miller, 1933) depend only weakly on it if β is close to 90°. The problem of refining coordinates (37) was aimed, first and foremost, at matching the drifts of Venus and Earth orbits with Clemens data (7). This was the most difficult problem, especially for Venus. However, the applied approach is not undisputable since the Clemens data for Venus and Earth may not match with future, more correctly measured values. The new parameters are as follows:

$$v_0 = 307.58 \text{ km/s}; \quad \beta = 85.3°; \quad \lambda = 110°. \qquad (39)$$

Table 2. The values of χ_1, χ, $\Delta\rho_1$, $\Delta\Phi$, ρ_1 (0) and $\Phi(0)$ for planets and asteroids

Object	$\chi_1\times10^6$	$\chi\times10^6$	$\Delta\rho_1$	$\Delta\Phi$	ρ_1 (0)	$\Phi(0)$
Mercury	0.458	0.503	-0.111	0.256	-0.336160	0.0239164
Venus	0.297	0.295	-0.0891	0.179	-0.341958	-0.0200508
Earth	0.241	0.252	-0.0494	0.0996	-0.336653	-0.0153888
Mars	0.211	0.207	0.0416	-0.0879	-0.322148	0.00152113
Icarus	0.213	0.288	0.0934	-0.835	-0.126622	0.252409
Phaeton	0.251	0.392	-0.0557	0.775	-0.0785358	0.273138
Ra-Shalom	0.181	0.204	-0.00903	0.0272	-0.271055	0.110333
Aten	-0.625	-0.605	-0.0822	0.186	-0. 292079	0.0378032
Kruitni	0.202	0.219	-0.0770	0.265	-0.237501	0.142637
Apollo	0.0933	0.100	0.0576	-0.215	-0.203742	0.165100

Table 2 gives the values of χ_1 and $\chi = \chi_1 + \chi_2$ for these parameters and for the same celestial bodies that are listed in Table 1. The discrepancies $\Delta\rho_1$ and $\Delta\Phi$, as well as the initial values of the functions ρ_1 and Φ for $\varphi_0 = 0$ are also given here. The values of $\rho_1(0)$ and $\Phi(0)$ are given with six significant digits to ensure their suitability for rapid control of other characteristics presented in Table 2. Data (39) should be treated approximate. There are only two ways to improve their accuracy: (i) evaluate more precisely the quantities v_0, β, and λ by the Shtyrkov method upon refining the method itself; (ii) obtain more reliable experimental data on the values of O–C derived using radar techniques for a number of planets and asteroids with significantly different spatial orientations of the orbits. In the first case, we will be able to find the orbital drifts of any planets or asteroids and, moreover, properly estimate the quadrupole moment of the Sun. In the second case, it will be possible to determine more exactly both the values of v_0, β, and λ and the quadrupole moment of the Sun.

Table 3. The values of χ_1, $\Delta\rho_1$, $\Delta\Phi$ for various eccentricity e of the orbit

e	$\chi_1\cdot10^6$	$\Delta\rho_1$	$\Delta\Phi$
0.1	0.850	-0.118	0.251
0.2	0.468	-0.111	0.255
0.3	0.346	-0.104	0.262
0.4	0.291	-0.0952	0.272
0.5	0.266	-0.0858	0.286
0.6	0.262	-0.0752	0.305
0.7	0.279	-0.0629	0.333
0.8	0.335	-0.0484	0.376
0.9	0.511	-0.0299	0.456

Table 3 gives the values χ_1, $\Delta\rho$, and $\Delta\Phi$ for various eccentricity e of the orbit. The parameters σ, Ω, ω, and i correspond to the orbit of Mercury. The calculations are performed for the initial data specified by Eq (39). Tables 2 and 3, Figs 7 and 8, and some other calculations results lead to the following conclusions.

1. The new algorithm for calculating the perihelion drift χ per one orbital cycle shows that the value of χ depends not only on the eccentricity e and the semi-major axis a (or σ) of the orbit, but also on its spatial orientation specified by the angles Ω, ω, and i.

2. If only the eccentricity e varies in the range from e = 0.1 to e = 0.9, the orbital drift χ_1 caused by the translational motion of the Solar System with respect to the ether firstly decreases, then becomes minimum between e = 0.5 and e = 0.6, and finally the further increase in the eccentricity results in the increase of the precession drift of the orbit (Table 3).

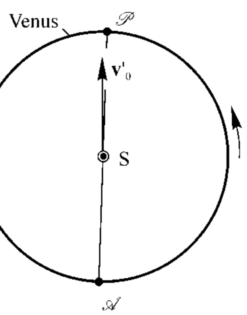

Fig 7. Comparative value of the vector $\mathbf{v}'_0$ and its direction relative to the Venus orbit

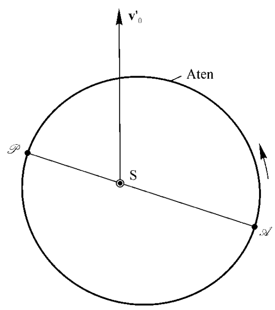

Fig 8. Comparative value of the vector $\mathbf{v}'_0$ and its direction relative to the Aten orbit

This conclusion contradicts Gerber formula (4) predicting that the drift increases monotonically with increasing eccentricity. However, the accuracy of the function of two variables is doubtful if it is known to be valid only at one point (Mercury) under the condition that the nonsphericity of the Sun is absolutely unimportant.

3. The drift χ_1 is maximum if the apse is orthogonal to the vector $\mathbf{v}'_0$, i.e., the projection of the translational velocity $\mathbf{v}_0$ onto the orbital plane. The vector $\mathbf{v}'_0$ for Venus (Fig.7) makes the angle of only 2° with the apse. This explains why the orbital drift remains within the values given by Eq (7) even for such a small eccentricity (e = 0.00677).

4. The orbital drift χ of Aten is maximal in magnitude and opposite in direction. The large value of χ is explained by two reasons: the angle between the apse and the vector $\mathbf{v}'_0$ is fairly large (72° according to Fig.8), while the eccentricity is relatively small (e = 0.183). The reverse drift ($\chi < 0$) is caused by the fact that the perihelion of this asteroid is situated on the left-hand side of the vector $\mathbf{v}'_0$, thus the absolute velocity at the perihelion turns out to be smaller than the velocity v'_0. Perihelion for each of other nine celestial bodies is situated on the right-hand side of the vector $\mathbf{v}'_0$, so these bodies pass their perihelion points with the absolute velocity exceeding v'_0. As a result, the drifts for all these bodies are positive (Table 2). It is known (Brillouin, 1970) that objects with reverse drift exist.

5. In Table 2, the total drift of Icarus amounts to only 52% of the vale calculated using the Gerber formula (5) (0.546×10^{-6}). This can explain the negative result by (Shapiro, 1965; Shapiro et al., 1968; Shapiro et al., 1971) on the determination of the quadrupole moment of the Sun using the difference between the observed value and the general-relativistic prediction. This difference turned out to be negative rather than positive, whereas the drift χ_2 caused by the solar oblateness is positive for Icarus (Table 1).

6. There are three considerations improving confidence in a new theory of planetary motion. Firstly, the very idea to attribute the additional drift of the orbit of Mercury, which is not explained by the classical celestial mechanics, to the translational motion of the Solar System through the static ether seems to be the simplest and the most natural as opposed to other known options. Secondly, nothing except operations demanded by the mathematical-formalism logic stands in the new theory between initial equation (15) and the final algorithm for calculating the precession drift of a planetary orbit. Finally, calculations based on the new theory highlighted the relation between two drastically different experimental facts within their actual accuracies, such as Miller experiments with the Michelson interferometer and the data of practical astronomy on the drifts of orbits of Mercury, Venus, Earth, and Mars. However, these arguments allow one to consider the theory only as fairly possible. Only observational astronomy is able to confirm o reject the new views on the problem of the precession drift of planetary orbits. The asteroid Aten is the most suitable object for these purposes. The sign of the classical residual O-C of the orbit of this celestial body should be found. The new algorithm predicts the reverse precession of Aten for any of the four variants of the Solar-System translational motion given by Eqs (35), (36), (37), and (39). Therefore, the conclusion of this anomaly in the motion of Aten will hold true even if further corrected observational facts will result in significant variations of the parameters given by Eq (39).

7. Confirmation of the predicted anomaly in the motion of Aten would imply that, firstly, that the absolute Newtonian space is real and, secondly, the source of classical-mechanics error is related to nullifying the translational motion of the Solar System. Probably the only correction necessary for the classical mechanics consist in replacing the Newtonian dynamical equation with Eq (15) where $\mathbf{v}$ stands for the absolute velocity of a point-like object.

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REFERENCES

Brillouin, L. (1970). Relativity Reexamined: New York: Academic Press.

Brumberg, V. (1991). *Essential relativistic celestial mechanics*: CRC Press.

Dicke, R. H. (1970). *Gravitation and the Universe*: Amer. Philosophical Soc.

Gerber, P. (1898). Die räumliche und zeitliche Ausbreitung der Gravitation.

Michelson, A., Pease, F., & Pearson, F. (1929). REPETITION OF THE MICHELSONMORLEY EXPERIMENT. *JOSA, 18*(3), 181-181.

Miller, D. C. (1933). The ether-drift experiment and the determination of the absolute motion of the earth. *Reviews of Modern Physics, 5*(3), 203-242.

Pitjeva, E. (2005). Relativistic effects and solar oblateness from radar observations of planets and spacecraft. *Astronomy Letters, 31*(5), 340-349.

Schlömilch, O. X., Witzschel, B., Cantor, M., Kahl, E., Mehmke, R., & Runge, C. (1866). *Zeitschrift für Mathematik und Physik* (Vol. 11): BG Teubner.

Shapiro, I. I. (1965). Solar rotation and planetary orbits. *Icarus, 4*(5), 549-550.

Shapiro, I. I., Ash, M. E., & Smith, W. B. (1968). Icarus: further confirmation of the relativistic perihelion precession. *Physical review letters, 20*(26), 1517.

Shapiro, I. I., Smith, W. B., Ash, M. E., & Herrick, S. (1971). General relativity and the orbit of Icarus. *The Astronomical Journal, 76*, 588.

Shtyrkov, E. I. (2005). *Observation of ether drift in experiments with geostationary satellites.* Paper presented at the Proceedings of the Natural Philosophy Alliance, pp201-205, 12th Annual Conference, Storrs CT.

Vavilov, S. (1956). Sobraniye sochineniy (The selected works). *Sobraniye sochineniy, , v. 4.*(Akad. Nauk SSSR, Moscow.).

