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An Elasto-Plastic Model for Analysis of Underwater Tunnels in a Strain-Softening Hoek-Brown Rock Mass

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ABSTRACT

This paper presents a method for analysis of underwater tunnels in axisymmetric plain strain conditions. The newly developed technique in this paper proposes a finite difference numerical method for calculating distribution of pore water pressure as a consideration for stress and strain around a circular tunnel excavated in a rock mass. The behavior of the rock mass around the tunnel is considered elasto-plastic in a strain-softening model. The model also takes into account the effects of increment in elastic strain within the plastic zone as well as the dilatancy angle. Seepage flow and secondary permeability due to hydraulic-mechanical coupling have also been considered in the plastic zone. Modification of previously tested models is used to determine a more accurate model for calculating distribution of pore pressure in the elastic zone. As governing equations do not have a closed form solution, a computer program has been prepared that is based on the proposed model. Accuracy and applicability of this method have been investigated with an example.

Key words: Underwater Tunnels, Seepage, Pore Pressure Distribution, Groundwater, Strain-Softening

INTRODUCTION

When a tunnel is excavated below the groundwater table, groundwater flows into the tunnel and the seepage force acts on the tunnel wall. Any element of rock mass is loaded on all sides by the force of such seepage as body forces. Any further excavation of the tunnel will affect permeability of the surrounding rock mass. These changes in permeability are affected by the state of stress, pore pressure and plastic deformations. Some cracks may be formed in the fractured rock mass by increasing the level of stress, while permeability of the rock mass is changed as the cracks widen and spaces are increased.

Much research has been done on fields of stress and deformation created by tunnel excavation and seepage flow in underwater tunnels. A study by Brown & Bray (1982) examined hydraulic-mechanical coupling in rock mass for analysis of underwater tunnels taking in to consideration changes of rock permeability in the plastic zone but the research failed to provide an accurate model for incorporating calculations for seepage in the a equations. Fahimifar &

Zareifard (2009) presented an analytical model that takes into account the hydraulic-mechanical coupling of rock mass and lining, the model considered effective stresses rather than total stresses by modifying the accurate seepage model of Kolymbas & Wagner (2007). The method assumed elastic strain as a constant within the plastic zone so that the effect of elastic strain was ignored in the plastic zone, such that the increment of plastic strain versus total strain was considered. Moreover, the dilatancy angle has been taken as constant in the strain-softening zone but its effects on deformations around the tunnel were disregarded.

This study presents an analytical model based on the method of Brown and Bray (1982) along with development of the accurate seepage model of Ming et al. (2010) Parameters of softening that are taken in to account in this proposed model are; variations of the dilatancy angle, increment of the elastic strain in the plastic zone and deviatoric plastic strain.

MODEL ASSUMPTIONS AND GOVERNING EQUATIONS

The computational model is considered by assuming the axisymmetric condition. This model involves incorporating different zones of the rock mass including elastic and plastic zones (the zone with strain-softening and the zone with residual strength).

The equilibrium equation for axisymmetric conditions of each element of rock mass in polar coordinates will be stated as equation 1: (Timoshenko & Goodier, 1994)

$$\frac{d\sigma_r}{dr} - \frac{(\sigma_\theta - \sigma_r)}{r} = 0 \qquad (1)$$

In axisymmetric conditions, the equation used to evaluate deformation-strain will be defined as below:

$$\varepsilon_r = -\frac{du}{dr} \qquad (2)$$

$$\varepsilon_\theta = \frac{-u}{r} \qquad (3)$$

$$\frac{d\varepsilon_r}{dr} = \frac{\varepsilon_r - \varepsilon_\theta}{r} \qquad (4)$$

In equation 1, σ_θ and σ_r denote major and minor principle stresses, respectively. In equations 2~4, ε_r and ε_θ denote radial and tangential strains respectively, while u is radial deformation

ROCK MASS BEHAVIOR

Nonlinear Hoek-Brown empirical strength criterion is used for rock mass determination. For a rock mass: (Hoek & Brown, 1980)

$$\sigma'_\theta - \sigma'_r = \left\{m\sigma'_r\sigma_c + s\sigma_c^2\right\}^{\frac{1}{2}} \qquad (5)$$

And for broken or plastic zone:

$$\sigma'_\theta - \sigma'_r = \left\{m_r\sigma'_r\sigma_c + s_r\sigma_c^2\right\}^{\frac{1}{2}} \qquad (6)$$

In the equations above, σ'_θ and σ'_r show major and minor effective stresses at failure respectively; σ_c is the uniaxial compressive strength of the intact rock material, m_r , s_r , m and s are strength parameters of intact and broken rock mass respectively.

The behavioral model of rock mass is the strain--softening model. The rock mass will show elastic behavior as long as the equation of principle stresses does not satisfy the strength criterion. Thereafter, the strength of rock mass will gradually incline to reach residual strength. Parameters of $\varphi(i)$, $\sigma_c(i)$, $m(i)$ and $s(i)$ are stated in terms of γ^p function (deviatoric plastic strain). In the plastic zone, it is assumed that the mentioned parameters can be described as a bilinear function of the deviatoric plastic strain (γ^p) .(Alonso et al., 2003)

$$w(i) = \begin{cases} w_p - (w_p - w_r)\dfrac{\gamma^p(i)}{\gamma^{p*}} & 0 < \gamma^p(i) < \gamma^{p*} \\ w_r & \gamma^p(i) > \gamma^{p*} \end{cases} \qquad (7)$$

It should be noted in this model that γ^p is the strain-softening parameter for controlling strength parameters of $\varphi(i)$, $\sigma_c(i)$, $m(i)$ and $s(i)$ in the strain-softening zone and is defined according to equation 8: (Park et al., 2008)

$$\gamma^p = \varepsilon_\theta^p - \varepsilon_r^p \qquad (8)$$

Where, ε_θ^p and ε_r^p denote tangential and radial plastic strains, respectively. In equations 2~4, ε_r and ε_θ denote radial and tangential strains respectively, while u is radial deformation

HYDRAULIC ANALYSIS

Permeability of Rock Mass

Permeability is correlated with deformations as a result of these hydraulic-mechanical couplings in rock mass. In this regard, equation 9 has been represented: (Ghadami , 2012)

$$K_r = K_o(1 + \eta\varepsilon_{vp}^2) \qquad (9)$$

In the equation 9, η is the parameter representing permeability variations of rock mass in the plastic zone, K_o is the initial permeability of rock mass and is ε_{vp} volumetric plastic strain $(\varepsilon_{vp} = \varepsilon_{rp} + \varepsilon_{\theta p})$.

Seepage Pattern and Pore Pressure Distribution

Patterns represented in this study for seepage in underwater tunnels are depicted in Fig. 1 (radial convergent seepage is considered for the plastic zone in the proposed model).

The equations provided for seepage patterns are based on the following assumptions:

-Permeability of rock mass is homogenous and isotropic.

-The flow is in steady state.

-The tunnel has a circular cross section with constant hydraulic potential.

This study has modified the model cited by Ming to produce a more accurate model to calculate pore pressure (Ming et al, 2010) in order to analyze seepage in underwater tunnels at the elastic zone. Seepage at the plastic zone was analyzed using Darcy’s law for radial seepage.

Elastic Zone

Equation 10 is equation suggested by Ming et al. for the use in cases when pore pressure is constant at the outer surface of the tunnel: (Ming et al, 2010)

$$P_w(x,y) = X(x,y) + \frac{P_a + Y(x,y)}{\ln\left[\frac{h}{r_o^2} - \sqrt{(\frac{h}{r_o^2})^2 - 1}\right]}\left(\ln\frac{x^2 + (y + \sqrt{h^2 - r}}{x^2 + (y - \sqrt{h^2 - r}}\right. \quad (10)$$

Where, r_o is outer radius of the tunnel, h represents depth of the tunnel from groundwater level, P_a denotes pore pressure at the outer surface of the tunnel, while $X(x,y)$ and $Y(x,y)$ are functions which are determined according to boundary conditions in underwater tunnels.

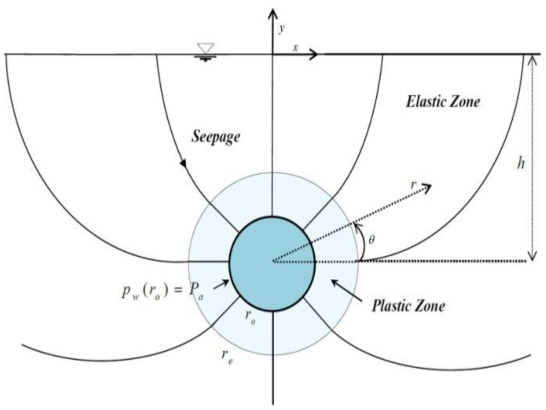


Fig.1 Seepage pattern in underwater tunnel

Since the above equation is capable of calculating seepage in different directions, replacing Cartesian coordinates with polar coordinates and applying boundary conditions, the equation for distribution of pore pressure is obtained as follows: (Ghadami , 2012)

$$\begin{aligned} P_w(r,\theta) &= (h - r\sin\theta)\gamma_w \\ &+ \frac{P_a - (h - r_o\sin\theta)\gamma_w}{\ln\left[\frac{(r_o\cos)^2 + (r_o\sin\theta - h + \sqrt{h^2 - r_o^2})^2}{(r_o\cos)^2 + (r_o\sin\theta - h - \sqrt{h^2 - r_o^2})^2}\right]} \times \\ &\left(\ln\frac{(r\cos\theta)^2 + (r\sin\theta - h + \sqrt{h^2 - r_o^2})^2}{(r\cos\theta)^2 + (r\sin\theta - h - \sqrt{h^2 - r_o^2})^2}\right) \end{aligned} \quad (11)$$

Plastic Zone

In the plastic zone around the tunnel, assuming a radial flow, distribution of pore pressure is obtained by using Darcy's low: (Fahimifar & Zareifard, 2009)

$$P_w(r,\theta) = \gamma_w q / 2\pi \int_{r_o}^{r} \frac{1}{rK_r(r)} dr + P_a - \gamma_w (r - r_o)\sin\theta \quad (12)$$

Where, K_r is permeability of rock mass in plastic zone and q is seepage rate. Equating the values of pore pressure at the elasto-plastic interface of the elastic zone in equation 11 with those in equation 12 produces the value of flow rate.

STRESSES AND DEFORMATIONS OF ROCK MASS

By using equations 1 and 5, the equilibrium equation for the plastic zone is written as below:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r} = \frac{\left[m(\sigma_r - P_w)\sigma_c + s\sigma_c^2\right]^{\frac{1}{2}}}{r} \quad (13)$$

Since equation 13 does not have a closed form solution, radial and tangential stresses are obtained at each step using a numerical solution (finite difference method): (Brown & Bray, 1982)

$$\sigma_r(i) = b - \sqrt{b^2 - a} \quad (14)$$

$$\sigma_\theta(i) = \sigma_r(i) + \left[\overline{m}(i)\overline{\sigma_c}(i)\sigma_r(i) + \overline{s}(i)\overline{\sigma_c}^2(i)\right]^{\frac{1}{2}} \quad (15)$$

Where:

$$a = \sigma_r^2(i-1) - 4c\left[\begin{array}{l}\frac{1}{2}\overline{m}(i)\overline{\sigma_c}(i)(\sigma_r(i-1)) \\ -P_w(i) - P_w(i-1)) + \overline{s}(i)\overline{\sigma_c}^2(i)\end{array}\right]$$

$$b = \sigma_r(i-1) + c\overline{m_a}(i)\overline{\sigma_c}(i)$$

(16)

$$c = \left[\frac{r_{i-1} - r_i}{r_{i-1} + r_i}\right]^2$$

$$\overline{w}(i) = \frac{1}{2}(w(i-1) + w(i))$$

In equation 16, w represents each of the strength parameters of s, m, σ_c and φ. In contrast with the Brown-Bray method, which is used to consider the elastic strain constant within the whole plastic zone, elastic strain increment is separately considered at each step of the calculation by the proposed model. Thus, the total strain would be divided into two parts, namely elastic strain and plastic strain.

$$\begin{Bmatrix}\varepsilon_r \\ \varepsilon_\theta\end{Bmatrix} = \begin{Bmatrix}\varepsilon_r^e \\ \varepsilon_\theta^e\end{Bmatrix} + \begin{Bmatrix}\varepsilon_r^p \\ \varepsilon_\theta^p\end{Bmatrix} \quad (17)$$

In the proposed model, the correlation between radial plastic strain increment $(\Delta\varepsilon_r^p(i))$ and tangential plastic increment, $\Delta\varepsilon_\theta^p(i)$, is given by equation 18: (Wang, 1996)

$$\Delta\varepsilon_r^p(i) = -K(i)\Delta\varepsilon_\theta^p(i) \quad (18)$$

$$K(i) = \frac{1 + \sin\varphi}{1 - \sin\varphi} \quad (19)$$

Where, φ denotes dilatancy angle. One solution runs calculations at the elasto-plastic interface assuming an elasto-plastic radius. Then it numerically solves plastic zone equations considering the values of stress and strain obtained at the elasto-plastic interface as initial values as long as the boundary conditions are met. The calculations are followed up until the elasto-plastic radius reaches a constant value.

VERIFICATION OF THE PROPOSED METHOD

As the proposed model does not have a closed form solution, the Utunnel (Underwater Tunnel) program is programmed by MATLAB software. This program is analyzed using the sample tunnel, while its obtained results have been interpreted and then compared with those from other models in order to verify it. A tunnel has been excavated from rock masses with parameters listed in Table 1. With respect to these specifications set out in Table 1, Brown and Bray analyzed this tunnel and cited the obtained results accordingly. The results were compared with those of the Utunnel program in Table 2. Fig. 2 depicts diagrams demonstrating the ground response curve and those of σ_θ and σ_r versus radius r calculated by the Brown & Bray (1982) method and the Fahimifar & Zareifard (2009) method, compared with those extracted by the Utunnel program.

The effects of incremental changes in dilatancy angle and elastic strain have not been considered at the plastic zone. Furthermore, Brown and Bray have utilized an inaccurate radial seepage pattern for hydraulic analysis. In spite of using the accurate seepage model of Kolymbas and Wagner (2007), the effects of incremental changes in dilatancy angle and elastic strain at the plastic zone have been neglected in the model offered by Fahimifar & Zareifard (2009). However, the Utunnel program has calculated the elastic strain increment at the plastic zone according to the dilatancy angle. It has also taken into account the effect of variations in dilatancy angle on performance of the tunnel.

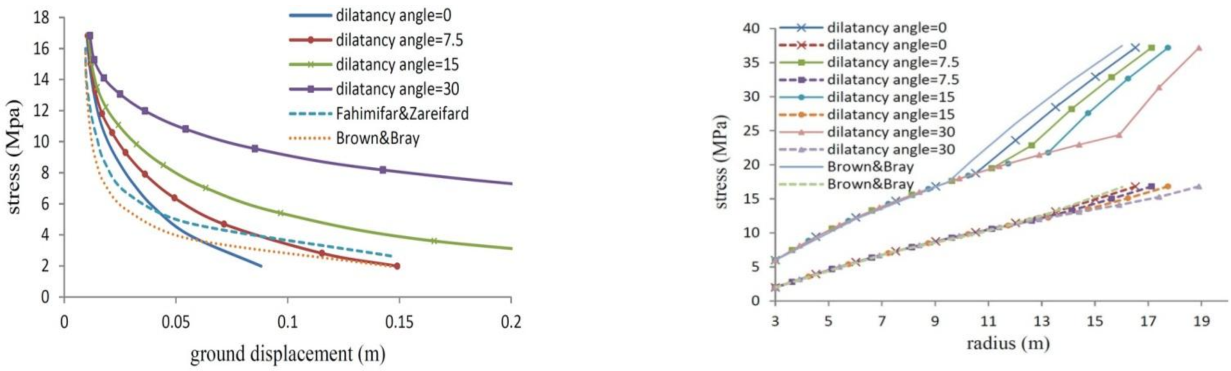


Fig.2 a. Ground response curve, **b**. Radial and tangential stresses at the plastic zone (plain lines denote tangential stress while dotted lines represent radial stress), h=300 m

Evaluations of the elasto-plastic radius increase according to an increase in the dilatancy angle regarding the effect of this angle and elastic strain increment at the plastic zone. Meanwhile, according to keeping the pressure of lining constant within the proposed model, it seems that the level of ground deformation before installation of the lining is considerably increased by raising the dilatancy angle.

The ratio between permeability at the plastic zone to initial permeability K_r / K_o is noticeably high from the excavated wall to the elasto-plastic interface. It is changed from 1 at the elasto-plastic interface to 16 at the excavated wall in the model proposed by Brown and Bray. The Utunnel program demonstrates that this ratio grows by increasing the dilatancy angle. This ratio remains constant at 1 for a dilatancy angle of 0°, while it is changed from 1 to 24 by increasing the dilatancy angle to 7.5°.

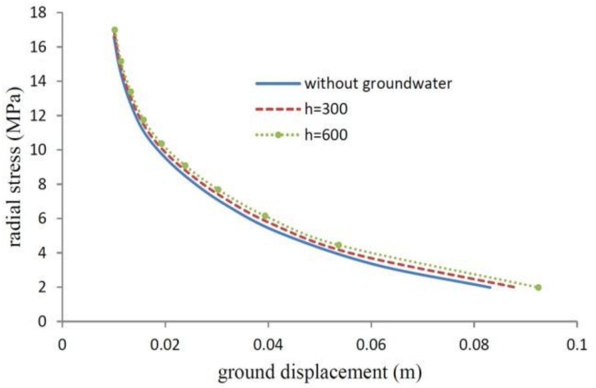
Table.1 Data of the tunnel analyzed by Brown & Bray (1982) method

Parameter	Value	Parameter	Value	Parameter	Value
Young Modulus (MPa) (E)	2000	m_P	0.65	m_r	0.2
Poisson ratio (υ)	0.2	s_P	0.2	s_r	0.0001
Initial stress (MPa) (P_o)	27	Internal friction angle (ϕ_P)	30	Strain-softening parameter (γ^*)	0.00375
σ_c (MPa)	40	Rock permeability (m/s) (K_o)	10^{-6}	Height of groundwater (m) (h)	300

Table.2 Comparison of the results of the Utunnel program with those of the Brown &Bray (1982) method

Parameter	**Brown & Bray model**	**Utunnel program**		
		$\varphi = 0$	$\varphi = \phi/4$	$\varphi = \phi/2$
Elasto-plastic radius (m)	16.024	16.5148	17.1237	17.7426
Radial stress in elasto-plastic radius (MPa)	16.73	16.8073	16.8122	16.8169
Tangential stress in elasto-plastic radius (MPa)	36.9	37.1927	37.1878	37.1831
Ground displacement at tunnel radius (m)	0.1434	0.0881	0.149	0.302

In Fig. 3, the effect of different conditions of groundwater level on the ground response curve, radial and tangential stresses have been demonstrated by keeping the dilatancy angle constant using the Utunnel program.



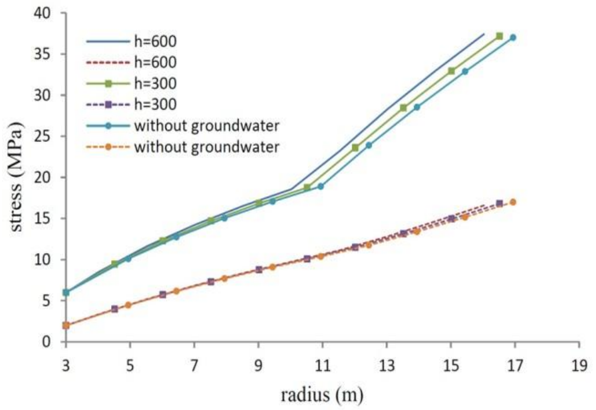


Fig.3 a. Ground response curves, **b**. Radial and tangential stresses at plastic zone (Plain lines denote tangential stress while dotted lines represent radial stress), $\varphi = cte$

As evident from Fig. 3, small changes in height of the groundwater level do not significantly affect evaluations for the elasto-plastic radius and ground deformation. Due to the high initial stress of this tunnel and good quality rock mass, limited variations in groundwater level will not have a considerable effect on the behavior of the surrounding rock mass.

CONCLUSIONS

The following main conclusions can be drawn from this study: In contrast with the method proposed by Brown & Bray (1982), this model made separate calculations for increments in elastic and plastic strain for each loop. Results demonstrated that by increasing the dilatancy angle, the plastic strain will be intensified at each loop, while raising the plastic strain will increase deformation of the rock mass and affect the elasto-plastic radius. Taking into account the separate calculation of elastic and plastic strains in the plastic zone, secondary permeability is correlated with the square of the plastic strain. Therefore, compared to the Brown and Bray model, the model presented in this study provides a more accurate criterion for considering hydraulic-mechanical interaction. Since both plastic and elastic strains are addressed in the plastic zone, the ratio of secondary permeability to initial permeability will be increased at greater dilatancy angles. Since application of the radial seepage model is inaccurate for shallow tunnels due to their significant error content, a combination of accurate non-radial model of Ming et al, (2010) and Darcy's radial model is recommended to model distribution of pore pressure around the tunnel. A new model has been made to determine pore pressure distribution at the elastic zone around the tunnel by using a modified version of the model of Ming. Moreover, a model has been offered considering hydraulic-mechanical interaction and secondary permeability for pore pressure distribution at the plastic zone around the tunnel. According to the results obtained from hydraulic analysis, both elasto-plastic radius and ground deformation are increased prior to installation of the lining by raising the groundwater level.

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