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# Comparison of lateral rear force between two and four wheel steering in a vehicle with steer by wire system

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**ABSTRACT**

One kind of steering systems in vehicles is Steering by Wire (SBW). Control and stability of this method are important issues. In fact the SBW system is very suitable for Four Wheel Steering systems. In this paper the SBW used for Four Wheel Steering system to take result. The effect of 4ws on turning is shown. When a 4ws system turns at low speed, the rear Wheels steer in the opposite direction as the front wheels, so that the maneuverability and parking of the system increases. At higher speed the rear wheels steer in the same direction as the front wheels, so that more stability and less lateral rear force are resulted. In this paper, a bicycle model is used for dynamic modeling for testing the stability and controllability. This model is built with the assumption theorem. Steer by wire is used for steering the rear and front wheels. The lateral rear force is estimated with a Hall Effect sensor.

**Key words**: SBW, 4WS, 2WS, lateral acceleration, yaw rate, bicycle model
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## INTRODUCTION

To reduce development time, improve design and miniaturization of complex systems, modelling and simulation is widely used in robotic vehicle engineering. This research focuses mainly on steering by wire systems and four wheel steering systems, modelling and comparing lateral rear force between two wheel steering and four wheel steering. A simulation of this model is done to check the systems stability. It is followed by vehicle handling characteristics of the four wheel steering system and its simulation. Finally a four-wheel steering model is tested and its praxis results are compared with the theoretical results.

Yih and Gerdes (2005) discussed a method with full vehicle state feedback. Accurate estimates of vehicle states are available from a combination of global positioning system (GPS) and inertial navigation system (INS) sensor measurements. Experimental results verify that with precise steering control and accurate state information, the handling modification is exactly equivalent to changing the front tire cornering stiffness. In another work they (Yih & Gerdes, 2004) developed a two-part observer structure based on linear models of the vehicle and tire. Observer estimates the vehicle states from measurements of steering angle and yaw rate.

A disturbance observer estimates the tire aligning moment, this estimation then becomes the measuring part of a vehicle state observer for sideslip and yaw rate. This approach to sideslip estimation also translates to vehicles equipped with electric power steering, since steering torque information can be obtained from the power steering system. Christopher D. Gadda, Paul Yih, J. Christian Gerdes (Gadda, Yih, & Gerdes, 2004) examined the benefits of incorporating vehicle dynamics modelling into the design of a diagnostic system for a steer-by-wire vehicle. The use of a model of vehicle dynamics improves the speed and accuracy of the diagnoses, by eliminating a significant source of disturbance input to the steer-by-wire system model.

A method for reducing the effects of modelling uncertainty on diagnostic system performance based on spectral fault characteristics is also presented. The discussed techniques are demonstrated on an experimental steer-by-wire vehicle. Shun-Chang Chang verified the chaotic motion of a steer-by-wire vehicle dynamic system. The largest Lyapunov (Chang, 2007) exponent is estimated from the synchronization to identify periodic and chaotic motions. Finally, a continuous feedback control method to control a chaotic vehicle handling and steering system.

Kim and You (2001) discussed an estimation method for side slip angle by using an unknown disturbance observation technique in 4ws passenger car system. First, a 4ws vehicle model with 3DOF is derived under the constant velocity and same tire's properties. The vehicle dynamic is transformed into the linear state space model with considering the external disturbance. Secondly, an unknown disturbance observer is introduced and its property which estimating the states of system without any disturbance information is shown. Kreutz, Horn, and Zehetner (2009) have presents two design strategies for an active rear wheel steering control system. The first method is a standard design procedure based on the well-known single track model. The aim of the feedback loop is to track a reference yaw rate in order to improve the handling behaviour. Unfortunately, a reasonable specification of the reference yaw rate proves to be a nontrivial task. A second approach avoids this drawback. The structure of the controller is regarded as a virtual mass-spring-damper system with adjustable parameters.

Due to the high abstraction level of this method, the controller parameters can be tuned intuitively. Experiments with a prototype vehicle illustrate the effectiveness of the two proposed methodologies. Hiraoka, Nishihara, and Kumamoto (2009) propose an automatic path-tracking controller of a four-wheel steering (4WS) vehicle based on the sliding mode control theory. The controller has an advantage in that the front- and rear-wheel steering can be decoupled at the front and rear control points, which are defined as centres of percussion with respect to the rear and front wheels, respectively.

Numerical simulations using a 27-degree-of-freedom vehicle model demonstrated the following characteristics: (1) the automatic 4WS controller has a more stable and more precise path-tracking capability than the 2WS controller, and (2) the automatic 4WS controller has robust stability against system uncertainties such as cornering power perturbation, path radius fluctuation, and cross-wind disturbance.

## DYNAMIC MODELLING

The following assumptions are made:
1) Vehicle is moving on the horizontal plane
2) Vehicle speed is very low
3) Longitudinal slippage neglected

We develop a vehicle dynamic model by neglecting some effects introduced by Suspension and tire deformation. In designing vehicles the two fronts and rear wheels are considered as single front and single rear wheels (bicycle model, Figure (1)). The inputs to the system are front wheel steering inputs and rear wheel steering inputs.

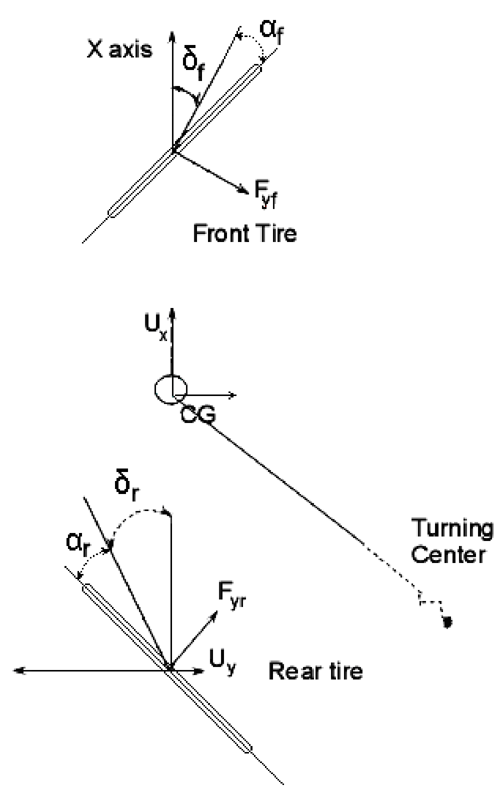


**Fig.1.** Bicycle Model (Free Body Diagram) and Velocities

The differential equations of motion can be written as

$$m\,a_y = F_y \tag{1}$$

$$I\,\dot r = F_y\,a \tag{2}$$

$$a_y = U_x r + \dot U_y \tag{3}$$

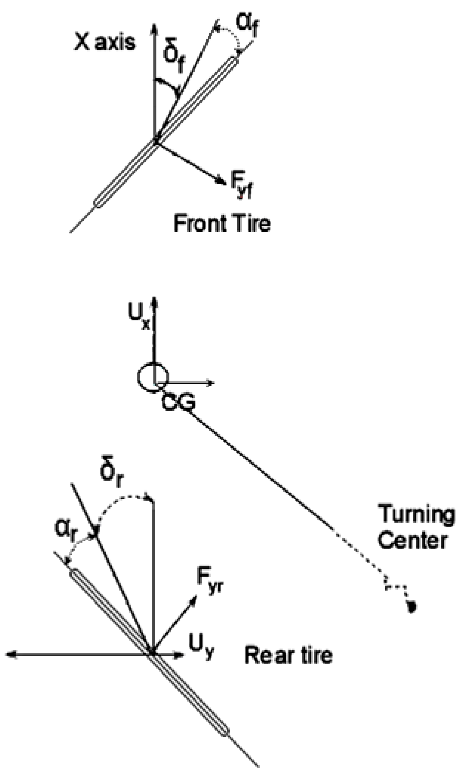


**Fig.2.** Velocities at front and rear tire

In Figure 2 can be seen that the equations may be written in a state space, where the system states are the yaw velocity (r) and the lateral acceleration. The inputs for this system are front and rear wheel steering angles. In Figure (1), the slip angles for the front and rear tire can be calculated as:

$$\alpha_F = \frac{v + ar}{V} - \delta_F = \beta + \frac{ar}{V} - \delta_F \tag{4}$$

$$\alpha_R = \frac{v + br}{V} - \delta_R = \beta + \frac{br}{V} - \delta_R \tag{5}$$

The equations of motion can be written as:

$$ma_y = F_{yf}\cos(\delta_f) + F_{yr}\cos(\delta_r) \tag{6}$$

$$I_z\dot r = aF_{yf}\cos(\delta_f) - bF_{yr}\cos(\delta_r) \tag{7}$$

$$F_{yf} = C_f\alpha_f = C_f\left(\frac{U_y + ar}{U_x} - \delta_f\right) \tag{8}$$

$$F_{yr} = C_r\alpha_r = C_r\left(\frac{U_y + br}{U_x} - \delta_r\right) \tag{9}$$

## STATE SPACE REPRESENTATION & STEERING SYSTEM RESPONSE

For making small angle approximations, the equations of motion can be put in state space form:

$$\begin{aligned} X &= AX + BU \\ Y &= CX + DU \end{aligned} \tag{10}$$

Here $X$ is the state space vector, $U$ as the input and $Y$ is the output.

$$\begin{bmatrix} \dot U_y \\ \dot r \end{bmatrix} = \begin{bmatrix} \dfrac{C_f + C_r}{mU_x} & \dfrac{aC_f - bC_r}{mU_x} \\ \dfrac{aC_f - bC_r}{I_z U_x} & \dfrac{a^2C_f + b^2C_r}{I_z U_x} \end{bmatrix} \begin{bmatrix} U_y \\ r \end{bmatrix} +$$

$$\begin{bmatrix} -\dfrac{C_f}{m} & \dfrac{C_r}{m} \\ \dfrac{aC_f}{I_z} & \dfrac{bC_f}{I_z} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix} \tag{11}$$

The step response of the bicycle model is unstable. The systems poles are (33.6328) and (42.3872). Since the poles are in the right half plane, the system is unstable. By placing poles for example at (-10+10i) and (-10-10i). The system can be made stable.

The above points are only generalized and poles can be placed at any desired location (according to the systems requirements). The characteristic polynomial for this closed-loop system is the determinant of (*sI-(A-BK)*).

Since the matrices A and *B*K* are both 2 by 2 matrices, so 2 poles for the system are resulted. By using full-state feedback, poles can be placed anywhere. A function place could be used to find the control matrix K, which will give the desired poles. The step response is stable. It can be seen from the plots that the behaviour of the two states are very similar. This is due to the fact that each state depends on both inputs. Any change in the inputs is distributed equally because of the vehicles symmetry conditions. This can be verified from matrices A and B in state space equations:

$$A = \begin{bmatrix} 42.00 & -1.80 \\ -1.80 & 34.02 \end{bmatrix} \qquad B = \begin{bmatrix} -1.00 & -1.10 \\ -0.90 & -0.99 \end{bmatrix}$$

## EXPERIMENTAL TESTS

A comparison between the lateral rear force of 2WS and 4WS systems is done by an experimental model. The main part of the system is consisted of stepper motors, stepper motor driver, rotary encoder, AVR microcontroller ATmega16 and Hall Effect sensor (Figure 3).

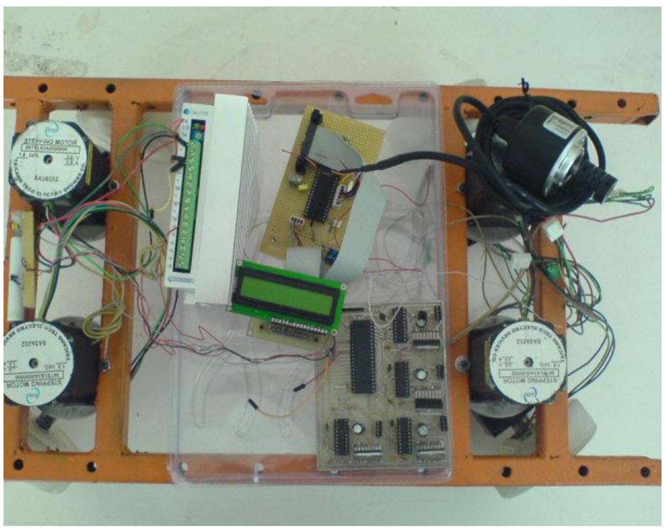

**Fig.3.** Model of 4ws

A Rotary encoder reads the users inputs and a microcontroller processes them for controlling the system. The steering equation between the front and rear wheels can be obtained from following equation:

$$K_S = \frac{\delta_r}{\delta_f} = \frac{\frac{Ma}{C_r L} U_x^2 - b}{\frac{Mb}{C_f L} U_x^2 + a} \tag{12}$$

The lateral rear force will be read with a Hall Effect sensor, by moving the magnet core. The magnetic field near the sensor will change. The output analogue voltage versus distance is shown in figure 4.

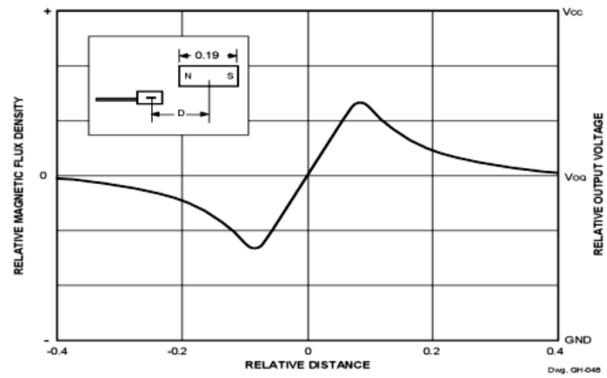


**Fig.4.** Hall Effect sensor

The experimental measurement of 2WS and 4WS rear force by the variable speed is shown in figure 5. Velocity is measured by pulse of controller and stepper motor angles. The force caused by turning of vehicle is measured by the sensor and then saved. Figure 4 shows that when the vehicle speed increases the lateral rear force also increase. The forces caused by 4WS are greater than 2WS at low speed because the turning radius of 4WS is smaller than 2WS. When the vehicle speed increased, at the critical speed the rear wheels turned in the same direction as front wheels. In this state the turning radius of 4WS system is greater than 2WS.

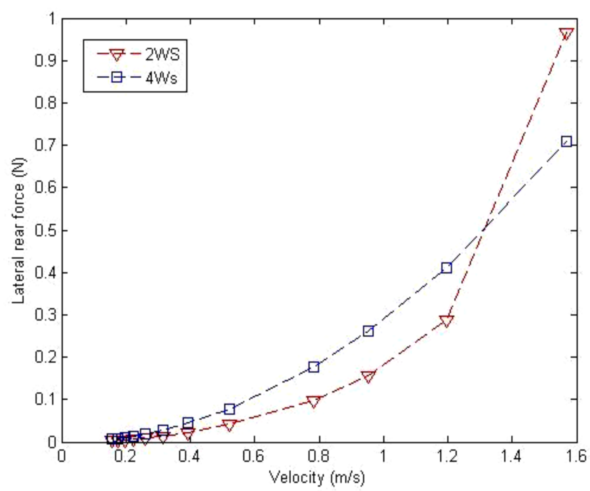


**Fig.5.** Experimentally obtained Lateral rear force versus velocity at 4WS and 2WS

The experimental and theoretical measurement of 2WS and 4WS rear forces by the variable speed is shown in figure 5. While increasing the velocity, lateral force will increase smoothly, and this rate can be seen in fig. 5. From comparison shown in fig.6, can be seen the higher force rate in 2WS which is greater than in 4WS.

Also by increasing the velocity, 2WS will create more lateral force than 4WS. In fact, the rotation in same direction as front wheels will cause the increasing in radius of rotation while this is the result of lateral force. In figure 5, the result of comparing between theoretical and experimental obtains of lateral force have been shown which is obviously the accuracy of results near together.

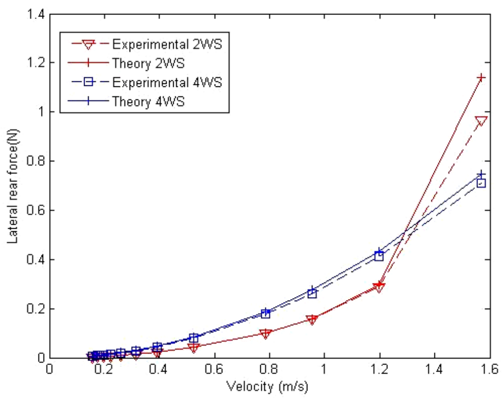


**Fig.6.** Lateral rear force versus velocity and Comparison with theoretical calculation.

## CONCLUSION

A four wheel steering system model was developed analytically and experimentally and dynamic analyses were shown in this paper. A four wheel steering scheme was discussed. Mathematical modelling was done for four wheel steering using a bicycle model where the front and rear steering has independent or dependant inputs. A simulation was shown. From which simulation can be resulted that the systems present configuration is unstable and can be stabled with pole place controller. A rear steering mechanism for 4WS system is proposed and installed for analysis. The vehicles steering angle, lateral acceleration, and lateral rear force and vehicle stability are discussed. From figure 4 and 5 can be taken that the lateral rear force in 2WS is smaller than in 4WS before a specific velocity in low level speed, but by increasing the velocity this will reverse with a high rate, which is an important result. The cross point of two curves specified in figure 4 shows that in this speed the angle of rear wheels in 4WS systems are similar to the angle of rear wheels in 2WS system. These results form the basis for more complex model development and mechanism design and can be valuable for future research.

## NOMENCLATURE:

| | |
|---|---|
| $a_r$ | Front tire distance from CG along longitudinal axis |
| $a_y$ | Lateral acceleration |
| $b$ | Rear tire distance from CG along longitudinal axis |
| $C_f$ | Front tire cornering stiffness |
| $C_r$ | Rear tire cornering stiffness |
| $F_y$ | Lateral force |
| $F_{yf}$ | Lateral front force |
| $F_{yr}$ | Lateral rear force |
| $I_z$ | Moment of inertia of vehicle about its yaw axis |
| $l$ | Vehicle wheelbase |
| $m$ | Mass of vehicle |
| $\dot{r}$ | Yaw rate |
| $U_x$ | Longitudinal velocity |
| $U_y$ | Lateral velocity |
| $\dot{U}_y$ | Lateral acceleration |
| $v$ | Lateral vehicle speed |
| $V$ | Longitudinal vehicle speed |
| $\alpha_f$ | Front tire slip angle |
| $\alpha_r$ | Rear tire slip angle |
| $\beta$ | Side slip angle |
| $\delta_f$ | Front steering angle |
| $\delta_r$ | Rear steering angle |

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