



Reality of Schrödinger's Cat

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ABSTRACT

The evidence that the probability interpretation is indispensable for the wave function has not been reasonably revealed since the early stages of quantum theory like the topic of Schrödinger's cat has been discussed from various viewpoints. Recently the Schrödinger equation has reasonably derived from the diffusion equation in accordance with the causality for the Newton mechanics, regardless of the de Broglie's hypothesis. In the derivation process, the problem of probability interpretation has been reasonably solved in relation to a wave function collapse, and moreover not only the evidence for a micro particle having a wave nature but also the evidence for validity of the de Broglie's hypothesis itself has been theoretically revealed. Further, the other fundamental problems having been unsolved for a long time are also reasonably solved. For example, it is theoretically revealed that such a single composite particle as a cluster molecule has a wave nature when it is composed of atoms smaller than about 770 numbers.

Keywords: Matter Wave, Diffusion Equation, Schrödinger's Equation, Wave Function Collapse, Minimum Time in Physics.

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INTRODUCTION

The Einstein's relativity and the quantum theory have been applied to solving problems unable to understand in the Newton mechanics. Accepting the constant principle of light velocity, the relativity was established by denying the absolute time-space valid in the Newton mechanics. On the other hand, the quantum theory has developed by accepting the de

Broglie's hypothesis unable to understand in the Newton mechanics from the beginning (De Broglie, 1923). We cannot thus understand the causality between the Newton mechanics and the quantum mechanics. In the quantum theory, such fundamental problems that the theoretical evidence has not been revealed have been still left in the unsolved state. Their subjects unrevealed are as follows:

<A> The theoretical evidence that a micro particle has a wave

nature.

 The theoretical evidence that the matter wave relation proposed by de Broglie as a hypothesis is really valid in behavior of a micro particle.

<C> The theoretical evidence that we should accept the wave function as an existence probability of a micro particle.

<D> The theoretical evidence that the quantum phenomena are expressed as a superposition of events in the past and the future.

<E> The theoretical evidence that the quantum phenomena of micro particle disappear from a certain larger size.

<F> The theoretical evidence that the incomprehensible behavior of a micro particle happens in the double slits problem.

Here, the present text reveals that the fundamental problems mentioned above <A> ~ <F> are reasonably solved by denying the density theorem of real time which is valid in the Newton mechanics. Before revealing matters, we first briefly discuss the diffusion problem in relation to the derivation of the Schrödinger's equation from the diffusion equation (Schrödinger, 1926).

Judging from the fact that a concentration profile of diffusion particles as well as a temperature profile in a material shows the parabolic law, Fick proposed the diffusion equation similar to the Fourier's thermal conduction equation (Fick, 1855; Fourier, 1822). After that, the diffusion equation has been accepted as Fick's second law unable to prove the theoretical validity. In that situation, recently the diffusion equation was reasonably derived from the mathematical theory of Markov process (Markov, 1960; Takahisa Okino, 2015).

WAVE NATURE OF MICRO PARTICLE

In accordance with the Markov process, the concentration $C(t, x, y, z)$ of diffusion particles in the time-space (t, x, y, z) satisfies the relation of

$$C(t + \Delta t, x, y, z) = \frac{1}{2} \{C(t, x + \Delta x, y + \Delta y, z + \Delta z) + C(t, x - \Delta x, y - \Delta y, z - \Delta z)\} \quad (1)$$

The Taylor expansion of the both sides of equation (1) yields

$$\begin{aligned} \frac{\partial}{\partial t} C &= D \langle \tilde{\nabla} | \nabla \rangle C \\ \text{for } D &= \frac{(\Delta r)^2}{2\Delta t} \quad \text{and} \quad \langle \Delta r | = (\Delta x, \Delta y, \Delta z) \end{aligned} \quad (2)$$

where $\langle \tilde{\nabla} | = -|\nabla \rangle^\dagger$ is defined here in accordance with the Hermite characteristic of the Dirac's bracket as a vector representation. Here, $\{\langle \Delta r | \nabla \rangle\}^2$ is used as $(\Delta r)^2 \langle \tilde{\nabla} | \nabla \rangle$ in the Taylor expansion because of the vector $|\Delta r \rangle$ considered to be isotropic in a local space and the divergence theorem (Takahisa Okino, 2022). For the elementary process of diffusion in a small local space isolated, therefore, the diffusivity expression is then obtained as

$$D = \frac{(\Delta r)^2}{2\Delta t} = \frac{\Delta r p}{2m} \quad (3)$$

for a sufficiently small time Δt , where m and p are a mass of a diffusion particle and the momentum (Takahisa Okino, 2013, 2022). It is confirmed that the diffusion equation (2) is valid as a moving coordinate system even if the effect of an interaction between a diffusion particle and the other micro particles around the diffusion particle itself is incorporated into the diffusivity. Further, equation (2) is transformable into the Fick's second law of the fixed coordinate system then (Takahisa Okino, 2015). Judging from the theoretical frame of physics, at this point, the diffusion equation should not be now called the Fick's second law except a historical situation because of the reasonable derivation.

On the other hand, using the relation $r_n p = n\hbar$ ($n = 1, 2, \dots$) of quantization condition for an electron moving on the n -th orbital of a radius r_n around the nuclei of hydrogen atom, Bohr proposed the atom model in accordance with the empirical equation of spectrum reported by Balmer (Balmer, 1885; Bohr, 1913).

It is physically considered that the relation $r_n p = n\hbar$ corresponds to the angular momentum $L = \sqrt{\langle \tilde{L} | L \rangle}$ of $|L \rangle = |r_n \times p \rangle$. In that case, using the notation of $\Delta r = r_n - r_{n-1}$ for " $r_0 = 0$ ", the relation of $|\Delta L \rangle = |(r_n + \Delta r) \times (p + \Delta p) \rangle - |r_n \times p \rangle = |\Delta r \times p \rangle$ yields $\Delta L = \sqrt{\langle \Delta L | \Delta L \rangle} = \Delta r p = \hbar$ ". For an orbital electron with the mass m_e , the relation $r_n p = n\hbar$ is equivalent to the relation of

$$\Delta L = \Delta r p = m_e v^2 \Delta t = \hbar \quad (4)$$

because of $v = \Delta r / \Delta t$ for a sufficiently small Δt (Takahisa Okino, 2022). After the Bohr's atom model, it was experimentally revealed that an electron has a wave nature (Thomson, 1927). The orbital electron having a wave nature of a wave length λ satisfies the relation of

$$\lambda = 2\pi \Delta r \quad (5)$$

At a moment when a moving electron on an orbital escapes from the atom under a certain condition, it is considered that the momentum $|p \rangle$ of electron is still conserved. As a result, equation (4) is also applicable to the electron in the state of translational movement because of the validity of $\Delta r p = \langle \Delta r | p \rangle$ then. It was thus revealed that the relation of $\Delta r p = \hbar$ is valid in an arbitrary moving electron. Further, the equipartition law indicates that the relation of

$$m_e (v_x^2 + v_y^2 + v_z^2) = m_e v^2 = \alpha (k_B T + \varepsilon) \quad (\text{for } \alpha = 3) \quad (6)$$

is valid in such a collective motion of electron as a free electron gas, where α , k_B , T and ε are the degree of freedom, the Boltzmann constant, an absolute temperature and a correction factor at $T = 0$ in relation to the uncertain principle, respectively. Substituting equation (4) into equation (6) gives the relation of

$$\alpha (k_B T + \varepsilon) \Delta t = \hbar \quad (7)$$

which is valid regardless of the electron characteristic in spite of the discussion about the electron itself (Takahisa Okino,

2022). In other words, equation (7) is acceptable as a relation of α and T regardless of a moving state of micro particle in the time interval Δt . It was, therefore, revealed that equation (7) obtained here is acceptable as a relation for a micro particle with the degree of freedom α . Using the equipartition law for a diffusion particle having a mass m and a degree of freedom α , equation (3) is rewritten as

$$D = \frac{\Delta r p}{2m} = \frac{1}{2m} \{m v^2 \Delta t\} = \frac{\alpha(k_B T + \varepsilon) \Delta t}{2m} = \frac{\hbar}{2m} \quad (8)$$

It was revealed that the relation of

$$\Delta r p = \hbar \quad (9)$$

is also valid for an arbitrary micro particle in an arbitrary state. Thus, a micro particle has no such a state of $p = 0$ in accordance with the well-known uncertain principle. On the other hand, when two micro particles coexist in a local space of the size Δr , it is necessary in principle for discriminating them to use a photon of wave length λ under the condition of $\lambda < \Delta r$. In case of a sufficiently small local space, the photon with a high energy value $E = \hbar c / \lambda$ (c : light speed) collides them then. As a result, they are disturbed by the high energy photon and we cannot then discriminate in principle them. We accepted the fact as an impossible principle of discrimination between micro particles (IDM principle) (Takahisa Okino, 2022). In an elastic collision problem between two micro particles A and B of the same kind with mass m , when the micro particle A collides with the micro particle B standing still, the impulses $f_{AB} \Delta t$ and $f_{BA} \Delta t$ of micro particles A and B in the collision time Δt are expressed as $f_{AB} \Delta t_A = -m \Delta r_A / \Delta t_A$ and $f_{BA} \Delta t_B = m \Delta r_B / \Delta t_B$ under the conditions of $\Delta r_A = \Delta r_B = \Delta r$ and $\Delta t_A = \Delta t_B = \Delta t$, respectively.

If we cannot discriminate between A and B, the correlation between $f_{AB} = -m \Delta r_A / (\Delta t_A)^2$ and $f_{BA} = m \Delta r_B / (\Delta t_B)^2$ resulting from the replacement of the suffix A and B of f yields $\Delta t_A = \pm i \Delta t_B$. Based on the present discussion, the IDM principle reveals that there is a minimum time t_0 as a real time in physics. For a mathematical time $0 \leq \Delta t \leq t_0$, we must thus accept the imaginary time $\pm i \Delta t$ in physics. In that case, the differential operator $\partial / \partial t$ of partial differential equation relevant to behavior of micro particles should be rewritten as $\mp i (\partial / \partial t)$ because of $\lim_{\Delta t \rightarrow 0} (\Delta / \pm i \Delta t) = \mp i (\partial / \partial t)$ for $0 \leq \Delta t \leq t_0$ in mathematics.

Judging from eigenvalues of differential operators, it was thus found that the operators $\partial / \partial t$ and $|\nabla\rangle$ in the Newton mechanics corresponds to the imaginary ones given by

$$\frac{\partial}{\partial t} \rightarrow i \frac{\partial}{\partial t} \text{ and } |\nabla\rangle \rightarrow -i |\nabla\rangle \quad (10)$$

in the partial differential equation relevant to micro particles (Takahisa Okino, 2022).

In relation to the subject <A> mentioned above, when we substitute equation (8) derived from the diffusion theory and equation (10) resulting from the IDM principle into the diffusion equation (2) and rewrite the notation as $C \hbar \rightarrow \Psi$, the wave equation of Schrödinger given by

$$i \hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \langle \tilde{\nabla} | \nabla \rangle \Psi \quad (11)$$

is reasonably obtained regardless of the de Broglie's hypothesis (Takahisa Okino, 2022).

The wave function Ψ becomes such a complex value function as $\Psi = \Psi_1 + i \Psi_2$ then (Takahisa Okino, 2013). We could thus reasonably derive the wave equation of Schrödinger from the diffusion equation relevant to behavior of micro particle as a diffusion particle. The matter gives theoretical evidence that an arbitrary micro particle yielding phenomena of self-diffusion has a wave nature, and vice versa. As a result, accepting the existence of t_0 gives a wave nature of micro particle. In fact, it is experimentally confirmed that a molecule as well as an atom has a wave nature (Arndt et al., 1999; Shimizu, 2001).

In relation to the subject , it was revealed that an arbitrary micro particle has a wave nature in the subject <A>. Therefore, equation (5) relevant to the wave characteristic is also valid for a micro particle. Eliminating Δr from equation (5) of a wave image and equation (9) of a particle image results in the so-called de Broglie's hypothesis given by

$$p = m v = \frac{h}{\lambda} \quad (12)$$

Judging from the theoretical frame of physics, at this point, equation (12) is now not a hypothesis but a basic equation proved theoretically in physics.

PROBABILITY INTERPRETATION OF WAVE FUNCTION

Based on the results of the subjects <A> and , an image of micro particle corresponds to a wave packet resulting from the wave function. For a phase velocity v_p and a group velocity v_g of wave, it is known that v of equation (12) corresponds to the group velocity v_g . In that case, equation (12) and the definition of group velocity show that the relation of

$$v_g = v = \frac{d v}{d(1/\lambda)} = \frac{d v}{d(m v / h)} = \frac{h}{m} \frac{d v}{d v} \quad (13)$$

is valid as a frequency $\nu = \nu_m$ of matter wave. Integrating equation (13) gives the energy E of wave packet given by

$$E = \frac{1}{2} m v^2 = h \nu_m \quad (14)$$

Judging from the reasonable transformation between the diffusion equation and the Schrödinger equation, the energy of matter wave expressed by the Schrödinger equation should be accepted as $h \nu_m$ of a wave packet similar to the photon. In other words, the difference between a matter wave and the light is thus only one between the frequencies ν_m and ν from the present viewpoint, neglecting the difference between each mass of them.

In relation to the subject <C>, it was theoretically revealed that the diffusion equation of a micro particle is reasonably transformed into the wave equation of Schrödinger, and vice versa. The concentration C of the diffusion equation means numbers of diffusion particles per a unit volume. The relation $C \Delta V$ in a sufficiently small volume $\Delta V = \Delta x \Delta y \Delta z$ means an existence probably of diffusion particle if C is normalized then. In relation to the subject <A>, the relation $C \hbar \rightarrow \Psi$ is used in the derivation process of the Schrödinger equation.

In that case, it is apparent that the relation $|\Psi|\Delta x\Delta y\Delta z$ relates to the existence probability of micro particle because of the complex value function Ψ . In the Newton mechanics, the wave energy is proportional to the square of wave amplitude given by $|\Psi|^2$. If we make $|\Psi|^2$ correspond to the kinetic energy of a moving micro particle in the wave packet, $|\Psi|^2$ has a maximum value at the point occupied stochastically by a moving micro particle in the wave packet because of the existence of t_0 . Judging from the existence probability $C\Delta V$ and the correspondence $C\hbar \rightarrow \Psi$, we can accept that $|\Psi|^2$ normalized as $\iiint |\Psi|^2 dx dy dz = 1$ gives an existence probability of micro particle. The derivation of Schrödinger equation from the diffusion equation revealed thus reasonably the evidence that the square $|\Psi|^2$ of wave function means the existence probability of micro particle.

In relation to the subject $\langle \mathbf{D} \rangle$, the IDM principle revealed in the world of micro particle that there is a minimum time t_0 denying the density theorem valid in mathematics. Since the existence of t_0 has been already incorporated into equation (11) as imaginary differential operators of equation (10), the Schrödinger equation (11) is exactly valid in the mathematical theory. The mathematical solution $\Psi(t, x, y, z)$ obtained as a wave function is also exact. In physics, however, note that the time t in the world of micro particle should be accepted as

$$t_j = jt_0 \quad \text{for an integer } j (= -\infty, \dots, -1, 0, 1, \dots, \infty) \quad (15)$$

resulting from the IDM principle. In principle, therefore, the mathematical solution $\Psi(t, x, y, z)$ should be thus accepted in physics as $\Psi_p(t, x, y, z)$ given by

$$\Psi_p(t, x, y, z) = A_j \Psi(t_j, x, y, z) + A_{j+1} \Psi(t_{j+1}, x, y, z) \quad (16)$$

for $t_j \leq t \leq t_{j+1}$

where the expansion factors A_j and A_{j+1} relate to a existence probability of micro particle. Equation (16) reveals that the physical solution $\Psi_p(t, x, y, z)$ is obtained in principle by the superposition of the mathematical solutions of $\Psi(t_j, x, y, z)$ at the past time $t = t_j$ and $\Psi(t_{j+1}, x, y, z)$ at the future time $t = t_{j+1}$.

The matter means that the conception itself of present time is nonexistent in the world of micro particle. The topic of Schrödinger's cat is one where the superposition of equation (16) is metaphorically taken up for a subject in the macro world having no conception of the minimum time t_0 . As a matter of fact, if the observation time t_{obs} necessary for understanding behavior of a micro particle is estimated as $t_{obs} = (n - m)t_0$ using integers m and n under the condition of $m \leq j < j + 1 \leq n$, the physical solution $\Psi_p(t, x, y, z)$ of equation (16) is then rewritten as

$$\Psi_p(t, x, y, z) = \sum_{k=m}^n A_k \Psi(t_k, x, y, z) \quad (17)$$

for $t_m \leq t \leq t_n$

A matter wave at a given time-space is thus essentially expressed by the superposition of $\Psi(t, x, y, z)$ itself in the past and the future because of the existence of minimum time t_0 . If

we observe the wave function $\Psi(t, x, y, z)$ at $t = t_p$ between $t_m \leq t \leq t_n$, the functional value of $\Psi_p(t, x, y, z)$ is determined from equation (17) and $|\Psi_p(t_p, x, y, z)|^2$ gives an existence probability of a micro particle in a local space $\Delta V_p = \Delta x\Delta y\Delta z$ for $\Delta x = x(t_n) - x(t_m)$, $\Delta y = y(t_n) - y(t_m)$ and $\Delta z = z(t_n) - z(t_m)$.

Judging from a value of $|\Psi_p(t_p, x, y, z)|^2$, we can understand whether a micro particle exists in the localized space ΔV_p or not. The matter discussed here means that the functional value of $\Psi_p(t_p, x, y, z)$ corresponds to a wave function collapse (WFC) happened at $t = t_p$ between $t_m \leq t \leq t_n$. It is considered that the WFC happens when the wave function $\Psi(t, x, y, z)$ is influenced by a variation of physical system composed of the wave packet and a medium caused by such an extrinsic factor as a collision with the other particle and/or a photon, and so on. In addition, when the WFC has happened between $t_m \leq t \leq t_n$ in the wave packet having an energy value $h\nu_m$, subsequently the wave packet of particle image between $t_m \leq t \leq t_n$ propagates again for $t \geq t_n$ as a wave packet of wave image with an energy value $h\nu'_m$ because of the momentum $p \neq 0$ in relation to $\Delta rp = \hbar$.

LIMIT SIZE OF MICRO PARTICLE HAVING A WAVE NATURE

It has been experimentally revealed that not only such a micro particle as an electron having no internal structure but also such an atom having an internal structure has a wave nature (Shimizu, 2001). This indicates that a single atom is also acceptable as if it has no internal structure. It has been also experimentally revealed that a single molecule has a wave nature (Arndt et al., 1999). The matter indicates that such a composite micro particle is acceptable as a single micro particle like a micro particle having no internal structure. In other words, when the IDM principle is applicable to a micro particle, the micro particle has a wave nature even if it is composed of some micro particles. Further, the equipartition law shows that the degree of freedom relevant to a single molecule depends only on composing atom numbers regardless of the atomic structure.

The above situation is acceptable as if the conception of mass point model in the Newton mechanics is also valid in the quantum mechanics. Here, the IDM principle indicates that the wave nature of such a micro particle as a cluster molecule or a metal fine particle disappears when its size is larger than a certain one. For each of micro particles having a wave nature, however, we have never theoretically discussed its limit size having a wave nature in the history of quantum mechanics.

In relation to the subject $\langle \mathbf{E} \rangle$, equation (7) is valid in a sufficiently small Δt because of the validity of $v = \Delta r/\Delta t$ for $\Delta t \rightarrow 0$. On the other hand, the IDM principle revealed that the mathematical density theorem is not valid between $0 \leq \Delta t \leq t_0$ in physics. Rewriting Δt as $\Delta t \rightarrow t_0$, it is physically acceptable that equation (7) is rewritten as

$$t_0 = \frac{\hbar}{\alpha(k_B T + \varepsilon)} \quad (18)$$

Here, the mathematical density theorem is valid in the limit of $\alpha \rightarrow \infty$ and the quantum effect disappears then. Since the minimum value of α is physically $\alpha = 3$, the relation of $t_0 \leq \hbar/3(k_B T + \varepsilon)$ is valid. If we use tentatively a room

temperature $T = 290$ and $\varepsilon = 0$, the relation of

$$t_0 \leq 8.79 \times 10^{-15} \text{ s} \quad (19)$$

is approximately valid. Recently the value of minimum time t_0 was investigated by applying the Boyle-Charles's law and the Avogadro's law to the motion of gas molecules (Takahisa Okino, 2022). As a result, the value t_0 was then approximately evaluated as

$$1.14 \times 10^{-17} \text{ s} \leq t_0 \quad (20)$$

in accordance with experimental results (Lévi, 1927) and also in relation to results (Caldirola, 1980).

Equations (18), (19) and (20) give the approximate relation of

$$3 \leq \alpha \leq 2.32 \times 10^3 \quad (21)$$

because of $\hbar/(k_B T + \varepsilon) = 2.64 \times 10^{-14} \text{ s}$ in the present case. When such a single micro particle as a cluster molecule and a metal fine particle is composed of n atoms, equation (21) shows that the micro particle composed of atoms smaller than about 770 numbers has a wave nature, judging from the correlation between α and n . In addition, a wave nature of the fullerene C_{60} is experimentally confirmed (Arndt et al., 1999).

DOUBLE SLITS PROBLEM OF ELECTRON

Before discussing double slits problem of a micro particle, behavior of a photon is investigated here. The wave equation of a propagating speed v is expressed as

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 (\nabla^2 \psi) \quad (22)$$

In general, the wave phenomena expressed by equation (22) correspond to a shape variation of the single harmonic motion as a collective motion of medium having happened subsequently in the physical field concerned, for example a water surface wave, a sound wave, and so on.

A light wave is also expressed by equation (22). Nevertheless, a light wave propagates even in the vacuum having no medium. The matter shows that a light itself has also a translational motion as a particle image. Here, note that even if we substitute the imaginary differential operators of equation (10) into equation (22), the wave equation (22) is valid as it is. Further, a photon relates directly to the IDM principle. Therefore, using the mathematical solutions $\psi(t_j, x, y, z)$ and $\psi(t_{j+1}, x, y, z)$ based on the existence of minimum time t_0 , the physical solution $\psi_P(t, x, y, z)$ of equation (22) should be accepted as

$$\psi_P(t, x, y, z) = B_j \psi(t_j, x, y, z) + B_{j+1} \psi(t_{j+1}, x, y, z) \quad (23)$$

for $t_j \leq t \leq t_{j+1}$

similar to equation (16), where the expansion factors B_j and B_{j+1} relate to the existence probability of a photon at a given time-space (t, x, y, z) between $t_j \leq t \leq t_{j+1}$. It is thus acceptable that the physical solution $\psi_P(t, x, y, z)$ corresponds to the WFC of $\psi(t, x, y, z)$.

In a similar way to equation (17), the physical solution $\psi_P(t, x, y, z)$ should be also rewritten as

$$\psi_P(t, x, y, z) = \sum_{k=m}^n B_k \psi(t_k, x, y, z)$$

$$\text{for } t_m \leq t \leq t_n \text{ and } m \leq j < j+1 \leq n \quad (24)$$

For the double slits problem using a set of slits A and B and a screen, it is generally accepted that the interference fringe is formed by the interference effect of the wave functions having gone through the double slits A and B. Judging from the WFC discussed in the previous section, the light having gone through the double slits has no WFC until reaching the screen. As is known as a photoelectric effect, however, the light wave becomes a photon of particle image on the screen resulting from a focusing of energies in a local space $V_P = \Delta x \Delta y \Delta z$ because of the WFC expressed by equation (24). Subsequently, the photon is scattered by the screen surface and propagates again as a wave image, or disappears in the screen material giving energy.

In addition, when a light in a medium rush into a vacuum state, the WFC given by $\psi(t, x, y, z) \rightarrow \psi_P(t, x, y, z)$ happens between $t_m \leq t \leq t_n$ and then the light propagates as a photon of particle image in the vacuum for $t \geq t_n$ because of having no wave medium.

It is thus considered that the wave image of light corresponds to the wave function $\psi(t, x, y, z)$ itself while the wave function $\psi_P(t, x, y, z)$ caused by the WFC corresponds to the photon of particle image between $t_m \leq t \leq t_n$.

In relation to the Einstein's relativity and quantum theory unable to understand in the Newton mechanics, the relativity was established by denying the absolute time-space valid in the Newton mechanics resulting from accepting the constant principle of light velocity. The causality between the Newton mechanics and the relativity has been thus understood. Even if a moving photon having no mass is curved by a strong gravity field in the relativity theory, there is no room for incorporating the Newton's laws into the matter because of denying the absolute time-space. On the other hand, the existing quantum theory was established by accepting the de Broglie's hypothesis unable to understand in the Newton mechanics from the beginning. The causality between the Newton mechanics and the quantum theory has never been thus revealed since the establishment of quantum theory. In that situation, recently the quantum theory has been for the first time reasonably established in accordance with the causality for the Newton mechanics, where the density theorem of real time valid in the Newton mechanics has been denied by accepting the IDM principle in the world of a micro particle (Takahisa Okino, 2013, 2022).

Here, the present author thinks that the double slits problem accepted for a long time as an incomprehensible matter in the existing quantum theory is caused by not understanding the causality for the Newton mechanics. In the existing quantum theory established by accepting a concept of matter wave from the beginning, it seems that the wave function Ψ having both a particle image and a wave image is accepted in the Schrödinger equation. The new fundamental theory of quantum mechanics mentioned above indicates that the wave function Ψ has a wave image only as far as the WFC does not happen, in a similar to the wave equation (22) discussed in relation to a photon. The reason is as follows.

The transformation from the diffusion equation of micro

particle into the wave equation of Schrödinger corresponds to a change of concept from a particle into a wave. In fact, $|\Psi|^2$ means a probability distribution extended in a whole wave packet, which is not accepted as a particle image. In relation to the above matter of a photon in the relativity, we cannot incorporate the gravity conception in the Newton's laws again into the relativity, judging from the causality between the Newton mechanics and the relativity. In a similar meaning, we should not incorporate a particle image in the Newton mechanics into a wave packet again as far as the WFC does not happen, judging from the causality between the Newton mechanics and the quantum mechanics discussed above.

In relation to the subject $\langle \mathbf{F} \rangle$, the quantum theory indicates that a wave function of $\Psi(t, x, y, z)$ as a probability distribution of an electron before going through the slits becomes $\Psi(t, x, y, z) \rightarrow \Psi_A(t, x, y, z) + \Psi_B(t, x, y, z)$ just after having gone through the slits A and B as far as the WFC does not happen then. In that case, an interference fringe is formed by the interference effect between $\Psi_A(t, x, y, z)$ and $\Psi_B(t, x, y, z)$ on the screen. However, if we observe the electron just after having gone through the slit A at $t = t_p$ between $t_m \leq t \leq t_n$, the WFC happens through the observation. As a result, the WFC then indicates that $|\Psi_A|^2 \rightarrow (1 - \varepsilon)$ and $|\Psi_B|^2 \rightarrow \varepsilon$ for $0 \leq \varepsilon \leq 1$ is valid, and then the electron having gone through the slit A propagates again in the wave image for $t \geq t_p$ until reaching the screen. As a matter of course, an interference fringe is not formed on the screen then.

Behavior of a moving micro particle should be thus accepted a wave function $\Psi(t, x, y, z)$ as a probability distribution itself expanded in a whole wave packet. On the other hand, the wave function $\Psi_p(t, x, y, z)$ relevant to the WFC corresponds to a particle image because of the moving electron shown at a localized time-space (t, x, y, z) between $t_m \leq t \leq t_n$ in the wave packet.

DISCUSSION AND CONCLUSIONS

In the present work, it was reasonably revealed that the diffusion equation and the Schrödinger equation are transformable into each other. In that situation, it is well-known that a pollen particle shown in the Brown motion moves randomly. However, it is considered that a pure material composed of pollen particles does not show self-diffusion phenomena because of the present theory of $3 \leq \alpha \leq 2.32 \times 10^3$ for the degree of freedom. The diffusion theory

corresponding to the wave equation of Schrödinger should be thus limited only to the self-diffusion. In addition, the wave nature of micro particle will be reasonably revealed by investigating whether the self-diffusion of micro particles occurs or not in the experimentation.

The correlation between the evolution equations (2), (11) and (22) has been theoretically investigated applying equation (10) relevant to a characteristic of micro particles to the diffusion equation (2) and the wave equation (22) in the classical physics. The diffusion equation has been used for investigating behavior of random movement of micro particles unnecessary for discriminating between them. On the other hand, when the discrimination between micro particles is necessary, the imaginary differential operators of equation (10) should be substituted into the diffusion equation, which resulted in the wave equation of Schrödinger. Even if we substitute the imaginary differential operators of equation (10) into the wave equation (22) relevant to behavior of a light, the wave equation is valid as it is. We can understand a photon as a particle image, only when the WFC happens because of the existence of minimum time t_0 in a similar to a micro particle in the quantum theory. The matter gives evidence that the new quantum theory developed in accordance with the causality for the Newton mechanics has the validity in physics.

The relation $\Delta r p = \langle \Delta r | p \rangle$ corresponds to not only the translational motion of a micro particle but also its random movement resulting in the diffusion phenomena in a material.

The relation $\Delta r p = \sqrt{\langle \Delta r \times \vec{p} | \Delta r \times \vec{p} \rangle}$ gives theoretical evidence for a circular motion of a micro particle in such a shell structure as an atom and a nucleus and also in such a local space as a vacant lattice in a crystal material resulting in the well-known lattice vibration. The relation $\Delta r p = \hbar$ meaning a particle image in the quantum theory is thus essentially important for understanding behavior of a moving micro particle because of $p \neq 0$ for $\Delta r p = \hbar$.

Hereinbefore, the theoretical evidence of the subjects $\langle \mathbf{A} \rangle \sim \langle \mathbf{F} \rangle$ mentioned above were reasonably revealed in accordance with the existence of minimum time t_0 resulting from the IDM principle. From a viewpoint of theoretical frame of fundamental physics, it is thus extremely meaningful that the quantum theory is reasonably established in accordance with the causality for the Newton mechanics. Further, the existence of t_0 will give a great influence on fundamental theories in physics, for example the Big Barn theory, and so on.

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