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Gravitational analog of Bohr's theory for the solar system

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ABSTRACT

In this paper, the gravitational analog of Bohr's theory for the Solar System is presented by deriving explicit expressions for the gravitational analog of Bohr's quantum condition on angular momentum of a secondary (planet or satellite) revolving around its primary (the Sun or a planet) and Planck's constant h_G, in terms of known physical quantities of the System (the Solar or a Satellite System). The correspondence principle connecting atomic theory and gravitational theory is stated. It turns out that the ground state orbit of an electron in an atom corresponds to the Roche limit of the primary (defined in the text) in the gravitational case. The gravitational Planck's constant h_G when taken into atomic scale via the Correspondence principle gives an expression for the Planck's constant of the atomic scale. It has shown here that a one-to-one correspondence exists between the Planetary Distance Law and the Electron Orbital Distance Law.

Keywords: Roche limit-Planetary Distance Law, Gravitational Quantum Condition on Angular Momentum, Gravitational-Planck's constant, Correspondence Principle-Planck's constant in atomic scale, One-to-one correspondence between the Planetary Distance Low and Electron Orbital Distance Low, The role of resonance in generating discrete orbits in the planetary / Satellite system**.**

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INTRODUCTION

At the end of 19th century, J.J. Thomson discovered electrons. As electrons are negatively charged particles, and an atom as a whole is neutral, he imagined an atom to be a ball of positively charged medium in which the electrons are embedded like plums in a pudding. This atomic model came to be known as Thomson's plum-pudding model. Experiments performed by Rutherford and his colleagues soon revealed Thomson's plum-pudding model to be incorrect. They discovered that the positively charged medium in an atom is not disturbed uniformly as imagined by Thomson, but is concentrated in a small central volume called the nucleus. Taking the clue from the structure of the solar system, Rutherford imagined an atom to consist of positively charged nucleus around which negatively charged electrons are held revolving due to Coulomb's electrostatic force of attraction, just as planets are held revolving around the sun due to Newton's law of universal gravitation. This came to be known as Rutherford's solar system model of an atom. Soon, it was discovered that the Rutherford's model suffers from stability problem, in that, an accelerated electron should emit electro-magnetic waves and lose its energy and should slowly spiral in, ultimately falling into the nucleus. However, the fact is that the atom is stable. To overcome this difficulty and to keep an atom stable against its collapse as observations show, Bohr put quantum condition on the angular momentum of an electron revolving around the nucleus to generate the stationary orbits for electron in an atom. The Bohr-Rutherford's atomic model has been derived from the analogy of the structure of the solar system, and since then the questions have been: can we have the corresponding quantum condition on the planetary/satellite orbits? As such, planetary orbits are already quantized as revealed by Planetary Distance Law discussed in the text. Can the solar system have the corresponding quantum theory? Is the Newtonian gravitational field equation quantized? In this paper, we try to find answers to these questions.

QUANTUM THEORY OF NEWTONIAN GRAVITATION

Roche limit is defined as the distance around a primary on entering which a secondary is disrupted into pieces. Alternatively, it is the distance around a primary within which primordial matter is not condensed into a secondary. It is given by

$$R \cong a \left(\frac{\rho_p}{\rho_s}\right)^{1/3} r_* \qquad (1)$$

Where ρ_p is the density of the primary under consideration ρ_s that of secondary, r_* is the radius of the primary and a is a pure constant called here Roche constant, as it arises from the physical concept of Roche limit (Rawal, 1981, 1984, 1986, 1989a, 1989b, 1989c). In the solar system, its value is 1.442, and in the satellite system, its value is 1.26. If $\rho_p / \rho_s \sim 1$, then the Roche limit (R) assumes the form:

$$R = ar_* \qquad (2)$$

If $\rho_p / \rho_s \neq 1$ the Roche limit is modified slightly. Studying the contraction of a solar nebula or sub-solar nebula through a step of Roche limit, (Rawal, 1981, 1984, 1986, 1989a, 1989b, 1989c) has derive the planetary / satellite law in the form,

$$r_n = r_* a^n \,, \qquad n = 1,2,3,4, \dots k \qquad (3)$$

Where the r_* here is the radius of the primary (the sun or a planet) and a is the corresponding Roche constant. This Planetary Distance Law is consistent with resonant relation (Rawal, 1981, 1984, 1986, 1989a, 1989b, 1989c) which represent stationary orbits.

It is clear from Eq (3) that the planetary/satellite orbits are quantized and that the quantization has come from the process of the formation of systems, which is consistent with resonant structure s in the systems indicating stationary orbits. A secondary can therefore, revolve around its respective primary only in certain orbits given by Eq (3) which are stationary or non-radiative orbits. For later use, one would like to express Planetary Distance Law in terms of Roche radius Eq (2) rather than in terms of radius of the primary Eq (3). Doing this, we get

$$r_n = Ra^{(n-1)} \,, \qquad n = 1,2,3,4, \dots k \qquad (4)$$

The Electro Orbital Distance Law is given by

$$r_n = r_0 n^2 \,, \qquad n = 1,2,3,4, \dots k \qquad (5)$$

Where $r_0 \equiv r_1$ (the ground state orbit of an electron)

$$\equiv r_1 \equiv \left(\frac{4\pi\varepsilon_0 h^2}{4\pi m_e Z e^2}\right) \qquad (6)$$

Where the symbols have their usual meaning. The Newtonian law of gravitational field between two masses M_* and m at a distance r is

$$F_G = G\left(\frac{M_* m}{r^2}\right) \qquad (7)$$

Where G is the universal constant of Newtonian gravitation and then Coulomb's law of electrostatic field between two charges Z_e and e at a distance r in,

$$F_c = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Ze^2}{r^2}\right) \qquad (8)$$

Where $1/4\pi\varepsilon_0$ is, Coulomb's constant, ε_0 being permeability of the medium. Corresponding to Bohr's quantum condition for the angular momentum of an electron in stationary orbits around the nucleus,

$$mvr = \left(\frac{h}{2\pi}\right) n \qquad (9)$$

We may write the gravitational condition for the angular momentum of a secondary in stationary orbits around its primary as follows,

$$mvr = \left(\frac{h_G}{2\pi}\right) f(n) \qquad (10)$$

Where h_G is gravitational plank's constant and $f(n)$ is an unknown function of n, other physical quantities have their usual meaning. Using,

$$\frac{mv_n^2}{r_n} = \frac{GM_*m}{r_n^2} \quad (11)$$

In addition, from expressions (4) and (11), we get

$$h_G \equiv 2\pi m\sqrt{GM_*R} \quad (12)$$

and

$$f(n) \equiv a^{\frac{(n-1)}{2}} \quad (13)$$

These are explicit expressions for h_G and $f(n)$ in terms of known physical quantities of the system under consideration. Note that h_G involves a, the Roche constant through Roche limit R which is connected with the formation of a planetary/satellite system, contraction of a solar/sub-solar nebula through a step of Roche limit which has a precise value and physical meaning consistent with the fact that the quantization of planetary orbits is, indeed, connected with the formation of the planetary/satellite system, and that $f(n)$ is not a function of n alone as is the case with Bohr's theory but involves a, which is a pure constant.

Hoyle.F (1960) [also see Babinet (1861)] pointed out that the present mass of the planetary system may represent only a very small fraction of the total mass of the material which could have been present at the time of the formation. This is because most of the planets consist of the material, which is quite rare by normal cosmic standard. If the planets were created from solar material, which consists predominantly of H (hydrogen) and He (helium), it is necessary to take into account the large fraction of accompanying light gasses.

In order to construct planets like Uranus and Neptune which are thought to consist to mostly of ices and have masses of the order of $15M_\oplus$ and $17M_\oplus$ respectively ($M_\oplus$ is Earth's mass) we require about $1000M_\oplus$ of solar material for each planets. Similarly, for terrestrial planets which are mostly rocky, we again require a mass as large as $1000M_\oplus$ at each orbit as found by Hoyle and Wickramasinghe (1968). Rawal (1984) and references mentioned therein) the amount of solar material in each ring that has gone to form a planet is $600M_\oplus$ and not $1000M_\oplus$ as shown by Hoyle.F (1960) and Hoyle and Wickramasinghe (1968) as the solar nebula has shed out more number of planetary rings than known to the above authors. According to this scenario, the value of m in the expression (12) of gravitational constant h_G is the same for all planets and hence for a system under consideration h_G is constant. Similar is the case with satellite systems, which came into existence from sub-solar nebula. If this is so, the scenario represents the condition of formation of the planetary system rather than its evolution. In this case, m , M_* and R of the system remain unchanged, h_G of the system is constant. However, presently different planets/satellite have different masses and, therefore, $h_G = h_G(m_s) \equiv h_{Gs}$ where m_s is the present mass of the secondary. Similar is the case with Roche limit also, in that, Roche limit around a primary depends upon the mass, m_s or density, ρ_s , of the secondary. If one wants, one may consider specific gravitational Planck's constant $h_{G\ \text{specific}}$, which can be defined as the gravitational Planck's constant per unit mass. To go over from Bohr's theory to its gravitational analog in the solar satellite system and vice-versa using equations (7), (8), (10), (12) and (13), we have the following correspondence:

$$\frac{1}{4\pi\varepsilon_0} \leftrightarrow G \quad (i)$$

$$h \leftrightarrow h_G \equiv 2\pi m(GM_*R)^{\frac{1}{2}} \quad (ii)$$

$$Ze^2 \leftrightarrow M_*m_e \quad (iii)$$

$$n \leftrightarrow f(n) \equiv a^{\frac{(n-1)}{2}} \quad n = 1,2,3,4,\dots k \quad (iv)$$

(14)

This gives

$$r_0 \equiv r_1 \leftrightarrow R \quad (15)$$

Here r_1 is the ground state orbit of an electron and R is the Roche limit of the primary. The Eq (15) shows that the ground state orbit of electron in atom corresponds to Roche limit of the primary. Using Eqs (14) and (15), comparing (4), (5) and (6), it is easy to note that the planetary orbits are given in the Roche limit $R = a\,r_*$, that r_* is the radius of primary, a is the Roche constant-a pure number, and the electron orbits are given in terms of,

$$r_0 \equiv (r_1\ \textit{the ground state orbit of an electron}) \equiv 4\pi\varepsilon_0 h^2/4\pi^2 m_e Ze^2$$

or

$$r_0 \equiv r_1 \equiv \frac{4\pi\varepsilon_0 h^2}{4\pi^2 m_e Ze^2} \quad (16)$$

the ground state orbit of an electron in an atom. The ground state orbit is the first (lowest) stable orbit for an electron in an atom. Similarly, the Roche limit is the first (lowest) stable orbit of a secondary (planet or satellite) going around a primary (the sun or a planet) in a planetary or satellite system.

This brings out one-to-one correspondence between the Planetary Distance Law and the Electron Orbital Distance Law, that is,

$$r_0 n^2 \leftrightarrow Ra^{(n-1)}, \quad n = 1,2,3,4,\dots ,k \quad (17)$$

That is,

$$r_n(\textit{atomic system}) \leftrightarrow r_n(\textit{planetary sattelite system})$$

(18)

Consider now the gravitational Planck's constant h_G the expression (12). Due to correspondence (14) between two theories-gravitational field and electrostatic field and the expression (15), we have, using Eq(12) , for an atom

$$h_G \equiv 2\pi \sqrt{\left(\frac{1}{4\pi\varepsilon_0}\right) m_e Z e^2 r_0}$$

$$h_G \equiv \left[\left(\frac{1}{4\pi\varepsilon_0}\right) m_e e^2 r_{0(hydrogen\ atom)}\right] \equiv h \qquad (19)$$

It is interesting to note that, although gravitational Planck's constant h_G given by the expression (12) is system dependent as discussed previously, when it goes to atomic scale, via the Correspondence Principle given by the expression as (14) and (15) , and becomes h of the quantum theory, it depends only on the charge and the mass of an electron (All electrons have the same mass and the charge), the ground state orbital radius of an electron in a hydrogen atom, as a matter of fact for all atom, irrespective of their different atomic number and the Coulomb's constant.

These are all universal constants, which keep the value of h to be constant for all atoms and assign it the status of a universal constant.

SUMMARY

So far, we had only Planck's constant h in micro-world, which is a universal constant, discovered by Max Planck in the beginning of 20th century. Now we have gravitational Planck's constant h_G in macro-world, which is system dependent, but which when transformed to atomic scale via the correspondence principle stated in the text becomes a universal Planck's constant because of the properties of the atomic world. The work presented here reveals that due to very nature of gravity-contraction of Solar or sub-solar nebula through steps of Roche limit and the resonance at work in the system, the ring shedding process is quantized giving rise to system of discrete orbits in the planetary/satellite systems. It turns out that the ground state orbit of an electron in an atom corresponds to the Roche limit of the primary in the gravitational case (planetary/satellite system). It is also shown that a one-to-one correspondence exists between the planetary distance law and the electron orbital distance law.

REFERENCES

Babinet, M. (1861). Note sur un point de la cosmogonie de Laplace. *Comptes Rendus Acad. Sci. Paris, 52*, 481-484.

Hoyle, F., & Wickramasinghe, N. (1968). Condensation of the planets. *Nature, 217*(5127), 415.

Hoyle.F. (1960). The formation of the sun and planets. *Quart. J. Roy. Astron. Soc. 1, 28.*

Rawal, J. (1981). Resonant structures in the solar system. *The moon and the planets, 24*(4), 407-414.

Rawal, J. (1984). Contraction of the solar nebula. *Earth, Moon, and Planets, 31*(2), 175-182.

Rawal, J. (1986). Further considerations on contracting solar nebula. *Earth, Moon, and Planets, 34*(1), 93-100.

Rawal, J. (1989a). Contractions of subsolar nebulae. *Earth, Moon, and Planets, 44*(3), 265-274.

Rawal, J. (1989b). Planetary distance law. *Earth, Moon, and Planets, 44*(3), 295-296.

Rawal, J. (1989c). Planetary distance law and resonance. *Journal of Astrophysics and Astronomy, 10*(3), 257-259.