



Theory of relativity: Analysis of Lorentz transformation and Lorentz factor

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ABSTRACT

In this paper, we intended to understand why and how the Lorentz transformations act. Knowing the structure of formula will help us to consider more about the fallacy of time dilation. Here, we will show that the space occupied by an object is not lengthened, but it is as a consequence of reference system motion respect to the object.

Keywords: Relativity, Time dilatation, Lorentz transformation

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INTRODUCTION

Already we exposed the falsity of arguments that lead to the fallacy of the dilation of time (Viladesau, 2018a, 2018b). We now have to expose the mathematical formulas that show that the time is not dilated, but that the duration of the vision path of the observed event image is enlarged. We will use the methodology of graphically representing, in as much as possible we can, the necessary developments to better make us understand. In addition, more than trying to get to obtain the expressions of the aforementioned formulas, we will follow the inverse process. We will try to interpret what its variables

tell us and that A. Einstein incorporated in his theory. We will rely on graphic descriptions with little mathematical support. We will call extension the values of the variables space e and time t that should be considered in the formula that govern the natural phenomena that are intended to be observed. We have chosen to give them this name considering that in the observation of an event E . We will first detect its apparition and then observe its duration, ie its extension. To get to find where the action of the Lorentz factor resides within the so-called transformation formulas, in its presentation and we will use the figured language that, together with the support of drawings, we have developed our research work.

AN INTERPRETATION OF THE FORMULAS OF THE TRANSFORMATIONS OF LORENTZ

The Lorentz transformations (or transformation) are coordinate transformations between two coordinate frames k and k' with coordinate systems (x, y, z, t) and (x', y', z', t') respectively that move at constant velocity relative to each other. The usefulness of Lorentz transformations formula described in (Lorentz, Einstein, Minkowski, Weyl, & Sommerfeld, 1952). The most common form of the transformation is expressed in space-time as

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

and

$$t = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{2}$$

for coordinates k and k' .

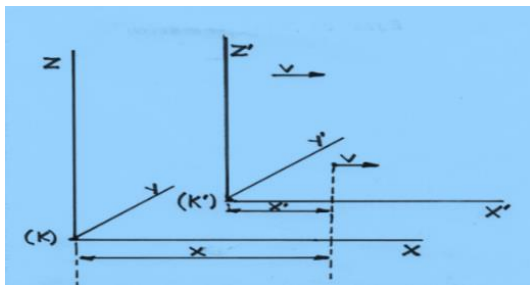


Figure 1

Here the variable v is relative speed of the system k' with respect to the system k . The interpretation of the usefulness of this Figure 1 is as follows: the situation defined by the axes k is regarded as a fixed reference frame (FRS) where a given event E seen from happening different situations that can be occupied by the axes k considered as a moving reference system (MRS). Regarding the information given in the aforementioned book we must comment that nothing is clear. That is why we added the following comment: We have presented Figure 1 only as information that is disclosed in the book of the referred Physical. However, we add of the following. We must not interpret that these coordinates serve to give an orientation of a body or an event, in three dimensions in space, but rather it is a way of situating a relative position of the events that occur in outer space. The x axis is define as direction of movement. However the z axis will couple to the measurement value and express extensions of events. It will be the measuring rod. The y axis is constant. We believe that the reader already knows the meaning of the inertial reference systems (IRS). In the book by the same author of this essay (Martínez, 2015) the concepts of position and situation are defined, as well as the concepts of inertial reference systems (IRS), fixed reference system (FRS) and moving reference system (MRS). To interpret the one and how the mentioned formulas are informing us, we have divided their study into the following two parts as, 1- Design of the events that occur in the observation of an event. 2- Vision of the event that is observed from the (MRS). First, we will apply

these two steps in obtaining the space formula. In the second we will apply them to the time formula.

SPACE FORMULA APPLICATION OF TWO STEPS MENTIONED

In our study, the first step is to perform and interpret the design of the events that occur in the observation of an event. We expose it in Figure 2, using figurative language to give reality to the process

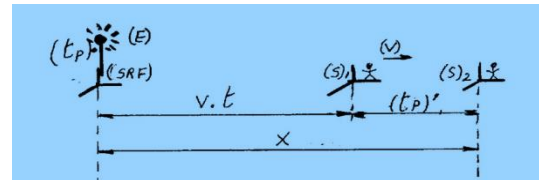


Figure 2

In sidereal space we will consider three situations connected to each other. It is about the following situations. (FRS) Fixed reference system. Situation on the occurrence of the event E in outer space. Here S_1 is denoted as situation in which the observer finds himself at the appearing of the event and S_2 is the situation from where the extensions of the event are observed. Although in Figure 2 the reference axes are represented with three dimensions, let us remember what we have already said. In fact only must consider two dimensions that are indicative. One is the direction of the x axis that will indicate the direction of relative movement. We will mention in the next paragraph the meaning of the variables: Own Time of extension t_p and own time corrected t'_p which appear in Figure 2. We assume that the reference axes system moving (MRS) travels at a constant v speed and rectilinear with respect to (FRS). (Recall that we are dealing with an Inertial Reference System (IRS)). The first consequence that we can draw from Figure 2 is that it is defining how to identify: the situation in which the event appears, the situation in which the observer is at this moment and up to where the moment will come when it will take place the observation of the event. The distance to be found S_1 of the (FRS) when the E is detected is represented by vt . The distance that will exist between the point where the (MRS) is located just at the moment of the occurrence of the event and the point in which it will be found when the observation is made will be worth

$$x - vt \tag{3}$$

We can see that this expression is equal to the numerator of the space formula

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \tag{4}$$

Looking at Figure 2, the second consequence we can obtain is that (t_p), will be

$$t'_p = x - vt \tag{5}$$

Regarding the value of the variable t'_p we must explain the following. Returning to Figure 2, we have drawn the variables t_p and t'_p Here t_p is particular time and represents the value of

the amplitude of the extensions, either in space x or in time t_p appeared in the (FRS) and t'_p represents the measure of the extension t_p measured from the one observation point of the (MRS). As we will see in the next paragraph $t_p < t'_p$.

VISION OF THE EVENT OBSERVED FROM THE (MRS) - INTERVENTION OF THE LORENTZ FACTOR

We had explained that this second part of the analysis would consist of the vision of the event. It is about being able to interpret how the extensions produced in the (FRS) are viewed and quantified from the (MRS). We will see that in order to realize this vision a grouping of mathematical variables is involved, called factor de Lorentz. The first thing that we comment is that the measurements that refer to a space x will be quantified in units of time. We will use the speed of light c as the unit of measurement. For example, the distance between two situations of outer space will be measured by indicating the time t that would take the speed of light c to travel the distance in which those two situations are located. We use Figure 3 to indicate what an observer already in his final position would see.

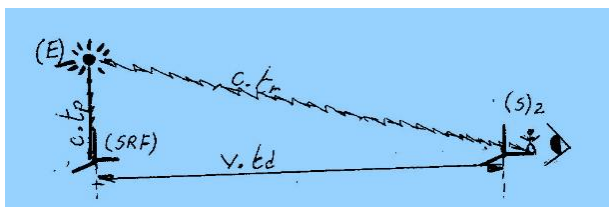


Figure 3

Call t_r the route of vision of the occurrence of the event, from the situation (FRS) to the final observation situation S_2 (Use Figure 2 to see this situation S_2). Being c the speed of light, the expression ct_r means the way of the vision of the image of the appearance of the event or also of the start of its extension. We call travel time t_d the time from which the event appeared to the situation in which observation is performed extensions. In order to evaluate the distance that will exist between the point where the extension has appeared and the observation point thereof, we will suppose a displacement that has occurred at a speed v and with a time t_d so the distance will be quantified as vt_d .

THE FACTOR OF LORENTZ-IDENTIFICATION OF THE DENOMINATOR OF THE FORMULAS

We need to interpret the function that meets the denominator that appears in the formulas we are studying. In order that the extensions of the event can be observed from a certain Situation of the (MRS) the following condition must be fulfilled.

CONDITION NECESSARY AND ENOUGH

This condition is announced as follows; That in the observation situation all the required information has been received. We can express it through

$$t_d = t_r \tag{6}$$

This indicates that to be able to see the image of the extensions, the time that will have elapsed from the starting point of the beginning of the observer to the point where the observation is made t_d must be equal to the time of the journey t_r of the vision of the image of the beginning of the extension of the event. By applying the Pythagorean theorem in figure 3 we have,

$$(ct_r)^2 = (ct_p)^2 + (vt_d)^2 \tag{7}$$

Demanding compliance with the condition $t_d = t_r$ it allows us to substitute t_r for t_d with what is obtained

$$(ct_d)^2 = (ct_p)^2 + (vt_d)^2 \tag{8}$$

From the above equation we have

$$(t_d)^2(c^2 - v^2) = (ct_p)^2 \tag{9}$$

$$t_d^2 = \frac{(ct_p)^2}{c^2 - v^2} \tag{10}$$

$$t_d = \frac{ct_p}{\sqrt{c^2 - v^2}} \tag{11}$$

We can transform the denominator in the following way

$$\frac{ct_p}{\sqrt{c^2(1 - \frac{v^2}{c^2})}} \tag{12}$$

by removing c we have,

$$t_d = \frac{t_p}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{13}$$

to the expression

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{14}$$

which allows us to relate the variables t_d and t_p is called Lorentz factor, and we will represent it by L . For what we can write $t_d = Lt_p$.

WHAT DOES THE LORENTZ FACTOR DO?

A mathematical deduction will make us see the intervention of this factor. In the formula

$$t_d = \frac{t_p}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{15}$$

Accordingly as we see the denominate of the above equation for $c > v$ always is less than unit. By dividing t_p by a figure less than one increases its size. In conclusion, t_d is greater than t_p . On the other hand, remember that when considering the obtaining of L we had imposed the condition $t_d = t_r$. Consequently t_r the time of the tour of the vision of the image of the extension of the event, will be greater than the value of

the extension t_p appeared in the (FRS). With what we have just explained, we believe that we can already appreciate the intervention made by the Lorentz factor within the aforementioned formula. What it does about t_d ? we must consider that it does about the numerator $(x - vt)$ of the formula.

WHERE CAN WE LOOK FOR THE CAUSE OF THIS EXTENSION?

In the case of inertial relative systems, we can already think that the cause of the enlargement of the vision of the image of the extensions in the route between two Inertial Reference Systems, we will find it in the displacement that exists between the moving reference system (MRS) to the fixed reference system (FRS). Figure 3 shows the three "time" variables that intervene in the observation of an Event. These variables are: the Own Time t_p and the Travel Time t_r of the extension image and the Displacement Time t_d of the observation point with respect to the vertical in which the event has appeared. Through calculations we have related these three variables and we have obtained a factor that relates the t_d to the t_p . We know that this factor is the so-called Lorentz factor.

$$t_d = Lt_p \tag{16}$$

With obtaining this factor we have been able to find a relation between the particular time of the event that is being observed and, therefore, it is a fixed time for this event, and the time elapsed until it has been decided to make the observation. We have left to analyze the variable t_r . In the time of the tour of the vision of the Image of the event, two causes intervene. A cause is the value of the extension itself, it will be its t_p . The other cause is the value of time t_d that the situation in which the observation has been made needs. Figure 4 aims to expose this double cause.

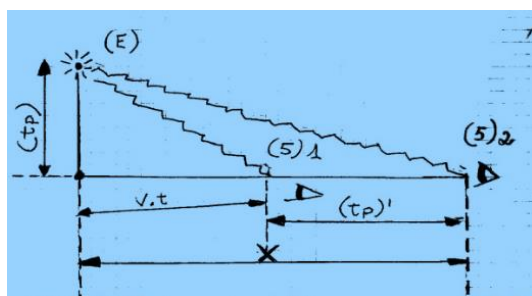


Figure 4

The development of a certain extension t_p corresponding to an event is observed more soon from the situation S_1 than from the Situation S_2 . The delay in the arrival of the image travel from the extension in the situation S_2 will make you appreciate more time. Another case would be if the observer were located in the same vertical as the E , that is, in the (FRS), then he would appreciate the start and end of the extension just when they happened. Observe that in this case the Lorentz factor would be worth one, since the relative velocity v is supposed to be zero. It may be evident but we believe that we should conclude with the following in mind : We measure the bodies and quantify them by for what we see we quantify what

we see that "encompass". We measure time and quantify it by for what we see. We quantify what we see that travel lasts of the arrows of the clock or the ticking of a pendulum. So, located in the Moving Reference System, considered this as the real life in which we are located and we move, we measure and operate with the data that we see and that come from another situation in the outer space.

We can conclude with the following consideration: Do not confuse the real magnitude of a certain event E that It occurs in a situation of outer space, with the vision of the magnitude of this event observed from another situation of outer space and that the observer is willing to operate with this value.

FORMULA OF THE TIME- APPLICATION OF THE TWO MENTIONED STEPS

As we have done in the case of the space formula the first step is to perform and interpret the "Design of events", which we expose in Figure 5 using figurative language to give reality to the process

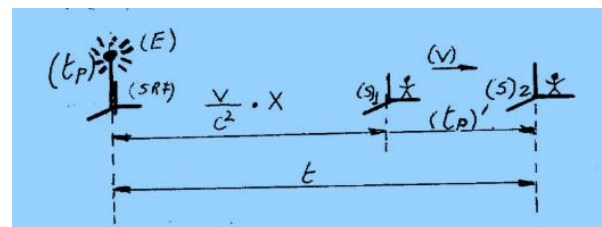


Figure 5

In this case t'_p represents the extension of the variable time. From this figure follows the following expression

$$t'_p = t - \left(\frac{v}{c^2}\right) x \tag{17}$$

that reminds us of the time formula

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{18}$$

We repeat the same as explained in the case of formula space and conclude that t'_p will be something bigger than t_p . Doubt may arise when questioning whether the second member of the numerator: vx/c^2 is really a time t to be subtracted from the first member, which is considered a time. For this reason we comment on the following paragraph

CORRECTION FACTORS OF THE SECOND MEMBER OF THE NUMERATOR OF THE FORMULA OF TIME

In the perception of the image of an event between two inertial reference systems (IRS) with a speed v between them, we must keep in mind that we are using two types of completely different speeds in terms of nature and orders of magnitude. It is about two different physical magnitudes. We can say that the relative speed v between the two reference systems is between two bodies, or "containers" of possible physical phenomena, while the speed of transmission of the information corresponds to an electromagnetic wave with a speed c very above or at least very different from the speed v . It is obvious

that although these are two expressions that are related to speed, are different. They do not have the same physical nature. They should be treated as different. The speed of light c is always the same, remember that in previous essays we said that it is inherent to itself, it is a constant and with an order of magnitude infinitely larger than what is supposed to be given on the axis x , that is v . We can not compare or establish a relationship between two lengths or two times that are measured using different patterns of measuring speeds. They must be normalized in such a way that both use the same type of pattern. We use the speed of light c as a pattern. That is, 300,000 (km/s) as a speed unit. To make the referred conversions we must take into account the following criteria to follow: All lengths will be quantified using light unit (LU). This means that light units are the ones that would be consumed to move between two specific reference points. For example, we can write $x = k(\text{LU})$ meaning that a certain length x is found, or it would require $k(\text{LU})$ to reach it. The "light unit" is a measure of speed that is worth 300,000 (km/

s). That is, one (LU), in the vacuum, is always approximately equal to 300,000 (km/s). A relation such as (v/c) assigns a fraction of (LU) at a certain velocity v , since c is a fixed quantity that is taken as a unit, while v is the relative velocity between (IRS), different value in each case in particular. To quantify a length (l) we will apply the expression $l = e(v/c)$ This expression answers the question: A length e that has been traversed at speed v at what length l equals if the velocity was that of light c ? To obtain the displacement time t_d of a Moving reference system (MRS) on the axis x , operating with (LU), we must divide the space equation by the speed of light c . That is $t_d = l/c$ and how $l = e(v/c)$ and we get

$$t_d = \frac{l}{c} = (e) \frac{v}{c^2} \quad (19)$$

Expression of a time that we have seen written in the numerator of the formula of the transformation of time and that is what we wanted to demonstrate.

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