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# Special Theory , Gravity And New Millennium Theory

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**ABSTRACT**

Gravity and time are necessarily related to each other. Gravity affects the measurement of time. In contrast time too affects the measurement of gravity. Our imagination about the gravity is like this much, that the gravitational constant $G$ is a constant character. That is not quite right. Gravitational constant $G$ is a relative character by all accounts. As we know that the measurement of time is affected both by motion (special relativity) as well as by gravity too (general relativity). Now the question may arise. Whether the measurement of gravity may also is affected by time? The answer is yes. Gravity too is affected by time (motion)as well. Gravity is the function of time.

**Keywords**: Gravity, Special relativity, General relativity, time dilation, Kepler's laws, Plank quantities.

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## INTRODUCTION

The principle of symmetry plays an effective role in physics in the discoveries of new phenomenon and relationships. Dealings with symmetry considerations often lead physicists to new insights and discoveries. This is true for Newton's laws of motion, for the laws of thermodynamics, for the laws of quantum mechanics, for the electromagnetic theory and for the theory of relativity as well. As an example, we know that the electromagnetic theory was developed purely on the basis of symmetrical considerations. Historically J.C. Maxwell developed his electromagnetic theory in order to encompass the earlier experimentally derived laws of Ampere and Faraday (Maxwell, 1881) . If a changing magnetic field creates electric field (Faraday law) then according to Maxwell's speculations a changing electric field must create magnetic field and it turned out to be true. By similar analogy the symmetry principle is helpful in describing the relative aspect of gravity. Gravity affects time then on symmetrical grounds time too affects the gravity. Einstein's theory of relativity have completely revolutionized our concepts of time, length and mass. The second postulate of relativity toppled our traditional concepts of relative speed, time and length completely, whereas the first postulate tells us about the relativity of mass and enables us to bring mass and energy into a single picture. The first question that arose to me when I studied relativity at very introductory level in 1990, concerning to the relativity of mass. The question was if the mass of an object increases at high speeds then what will happen to the gravity of earth in case if our earth suddenly starts running at high speed? Gravitational field of earth definitely must would increases proportionally with the increase in mass. Another a very puzzling question that occurred to me exactly in the same days was related to the relativity of time. An event that occurs in a frame moving with velocity 'v' relative to another frame would take longer time. The question that stroke my mind was, what would happen if I jumped down of a mountain of a speeding earth in contrast if the same experiment is done with an identical earth either at rest or moving slowly the speeding earth? Definitely it would take a longer time for me that I would reach near the surface of speeding earth. This indicates that the gravitational field of the speeding earth has decreased and the increase in the earth's mass plays no role. In this article I want to declare that the theory of relativity as developed by Albert Einstein and that gives us exact results with experimental verifications is only an approximation and further can be extended by appending the principle of equivalence with special relativity. Now Let's start our business. Here we are given three postulates. 1-The laws of physics may be expressed in the same fashion in all accelerated frames (non-inertial frames) within an equivalent gravitational field. Thus an accelerated frame is officiating an equivalent gravitational field (The principle of equivalence). 2-The laws of physics are same for all inertial observers in relative motion w.r.t one another (The principle of relativity). 3-Speed of light is the limiting speed of the universe. The first postulate states that an accelerated frame behaves in the same manner as an equivalent gravitational field. Suppose that we have an idealized Einstein's elevator (Fig1) moving relatively upward with high speed i.e. $v = 0.9c$ and an idealized object inside the elevator also is moving with the same speed in the same direction. Thus the object is at rest in the elevator with a constant force $F_{\mathrm{o}}$ in the same direction. Since the speed of light is the limiting speed of the universe and nobody can move with the same speed, acceleration thus produced in the elevator is very small. Though in relativity a constant force does not produce a constant acceleration and indeed acceleration is the function of the time elapsed, the instantaneous acceleration thus produced in the elevator is small under the influence of applied force and thus the idealized object inside the elevator instantly will appear to fall with small acceleration. Similarly if a man is standing on a weighing scale placed inside the elevator, then instantly the scale will register a less reading under the influence of the applied force. Now consider the same Einstein's elevator (Fig.2) with the same relativistic speed, i.e. $v = 0.9c$ say, in the upward direction and is acted upon by the same constant force $F_0$ say in the upward direction.

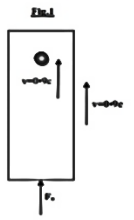

Fig 1

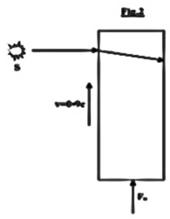

Fig 2

Let a light beam from a source $s$ enters the elevator. Since the elevator is very little accelerated under the influence of the constantly applied force $F_0$, the light entering the elevator thus will bend very small. The overall path of the light is nearly a straight rather than a curved one. Now since one cannot distinguish between the behavior of an accelerated frame and an equivalent gravitational field and also the accelerated frame is officiating an equivalent gravitational field, now we take into account the case of gravitational field. Suppose that our earth is moving relatively say with $v = 0.9c$ (Fig.3) and an object shown above the earth surface too is moving in the same direction with the same speed. Now doubt, the object moving with $v = 0.9c$ is at rest with w.r.t the moving earth, but according to the Einstein's postulate of general relativity (The principle of equivalence), the physics of this situation is same as that of the accelerated elevator, so the object shown above the earth surface is attracted by earth with small acceleration indicating that the gravitational field of earth is decreasing. Similarly a light passing this speeding earth (Fig.4) will bend very little by the earth's gravitational field.

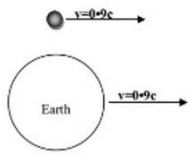


Fig 3

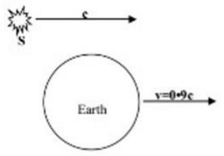


Fig 4

The overall path of light again is straight rather than a curved one as in the case of the accelerated elevator. The degree of the bending of light is the direct measure of the strength of the gravitational field. In order to check the prediction of the first postulate i.e. whether the prediction made by the first postulate may is true or absolutely wrong, we take into account the second postulate. This can be justified well by a simple example of school level physics for easy catch up to everyone. Let two observers Observer B and Observer A say, are agree to render their services in order to check the validity of the said prediction. Consider two planets one is our native earth and the second one is an imaginary planet, having the same rest mass as that of earth, the same volume (i.e. the same radius) as that of earth. Briefly a twin planet of our earth say. Now let Observer B is resting on earth and the earth is absolutely at rest in a gravity free region (Fig.5), whereas Observer A is resting on the imaginary earth (i.e. the twin planet) moving w.r.t the real earth in the $+ve$ x-direction with a speed $v = 0.866c$ say. Now let two objects are released at the same time from a height of 490 m above the surface of each planet. First we consider the case of real earth (Fig.6).

The time for the object to reach the real earth surface is about 10 sec. Now Let's consider the case of imaginary earth running with $v = 0.866c$ and the imaginary object shown too is moving with the same speed in the same direction a height of $490m$ above the imaginary earth (Fig.6).

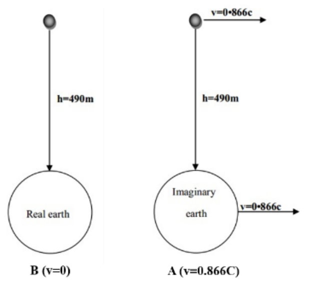


Fig 6

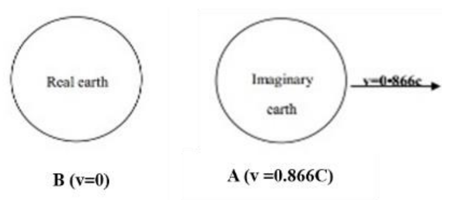


Fig 5

According to the principle of relativity the observer A resting on the imaginary earth will find the same time for the object with 10 seconds to reach the surface of this imaginary planet. Calculations made by A also will stand with the same result. Both the observers are agree about the same time for fall of objects to their respective planets. In the other words each observer is agree about the same value of g for their respective planets. The prediction of the first postulate has failed and the gravitational field is not decreasing. But here is an important point. Both the observers are observing different events, not the same event i.e. each observer is observing a proper event of their respective coordinate system. That is why the prediction of the first postulate is dying. The only reason that account for this failure. Now let's look at the same experiment with B point of view. The moving observer A claims that it takes exactly 10 seconds for the object to reach near the surface of his own planet (i.e. imaginary earth),while the rest observer B does not agree with observer A's claim, why? At this stage the important phenomenon of time dilation will come to play and B finds as $t_B = t_A/\sqrt{1-v^2/c^2} = 20\ sec$. Observer B claims that it takes 20 seconds for the object to reach the surface of imaginary earth. Now in 10 seconds the imaginary object covers only one fourth (122.5m) a distance as viewed by B and in the same time the object of his own frame has already reached the real earth (Fig.7).

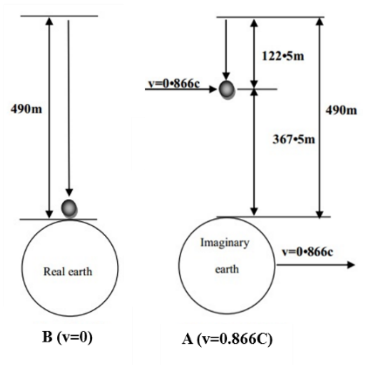


Fig 7

During this time (10 sec) the object above the real earth has reached its destination, whereas the object falling near the imaginary earth is in its (3/4) of its final resting place as viewed by observer B. Now what about the value of g for each observer? Observer B registers the time for the fall of imaginary object as, $t_B = 20$ /sec by his own clock whereas A measure $t_A = 10$ /sec by his clock and g with the point of view of each observer A and B is 9.8 $m/s^2$ and 2.45 $m/s^2$ respectively. Both observers A and B does not agree about the value of g. On the other hand A calculates four times greater than that for B. The principle of relativity has been seriously crushed. In order to save the relativity principle of any horrible consequence, the gravitational constant $G$ need to be modified by $(1\text{-}v^2/c^2)$ and $g_{(A)} = 4g_{(B)}$ or $g_{(A)} = 4G(1-v^2/c^2)M/R^2$ or $g_{(F)} = 4G(0.25)M/R^2 = GM/R^2$. In this case the laws of physics are same for both observers A and B in a relative motion. The object falls near the imaginary earth slowly i.e. with a small g as viewed by Observer B, confirms the prediction of the 1st postulate. The prediction made by the 1st postulate is completely consistent with the consequences of the 2nd postulate. Now Let's treat the same experiment with Observer A's point of view (i.e. the imaginary earth observer). Each observer finds that the objects falls near their respective coordinate systems exactly in 10 seconds i.e. Observer B says that the object reaches to his own rest frame (i.e. the real earth) in 10 seconds. Similarly Observer A too claims that it takes 10 seconds for the object to reach near the imaginary earth. Both the observers are right in their claim. But as Observer A diverts his attention from his own frame and focuses on the object falling near the real earth, he finds something wrong and disagrees with Observer B's claim. How? If Einstein is not wrong in his statement, then according to Einstein, the watch on Observer A's hand will run slowly w.r.t the Observer B's watch. For Observer B the period of fall to his own planet (i.e. the real earth) is 10 seconds. But Observer A registers a different time for the same event by his own clock as follow.

$$t_A = t_B\sqrt{1-v^2/c^2} = 5\ /\text{sec}$$

For Observer A the period of this fall is 5 seconds. Now when Observer A sees the object falling to his own rest frame (i.e. the imaginary earth) he finds that it has covered only one fourth (122.5m) a distance in this time and $(3/4)^{th}$ (i.e. 367.5m) of the way is remaining to its final destination(Fig.8).

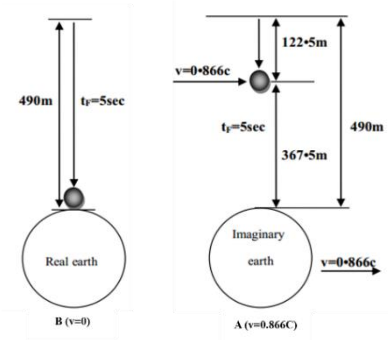


Fig 8

The value of g for the real earth from observer B and A is 9.8 m/s$^2$ and 39.2 m/s$^2$ respectively. Both observers are agree that the object falling near the real earth reaches first its surface than that reaching the imaginary one. But they are disagree about the strength of fields. The observers claims the natural strength (i.e. the original strength) for their respective planets (i.e. g = 9.8m/s$^2$) as in accordance to the relativity principle. Observer B claims that he is living in a natural field (i.e. original field, g = 9.8m/s$^2$), while Observer A is suffering of a decreased field. On the other hand Observer A disagrees with Observer B's statement and claims that he is enjoying the same natural field (i.e. g =9.8m/s$^2$) whereas Observer B is facing an increased field. Both the observers are really in a dilemma. Now both the observers decides to empower the check to a third neutral observer O say, watching the situation carefully from a rest point $P$ say in space (Fig.9).

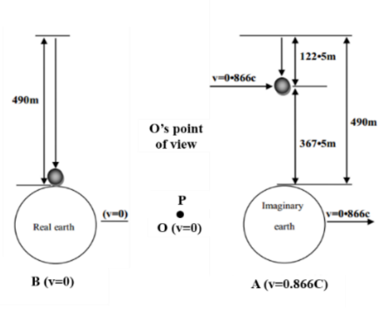


Fig 9

Now both the objects are released from a height of 490 m above their respective planets at the same time in front of Observer O. The real earth is at rest Observer O. The period of fall for Observer O is same as that for Observer B, i.e. t=10 seconds. In other words Observer O too is treating this event properly. The imaginary earth is moving with $v = 0.866c$ Observer O. The period of fall for the imaginary object will take a longer time for Observer O by an amount $1/\sqrt{1-v^2/c^2}$. This event is a dilated event for Observer O, i.e. $t_o = t_A/\sqrt{1-v^2/c^2} = 20\, sec$. Observer O at the same time is playing a dual role. The fall of the object near the real earth is a proper event for Observer O, whereas the case of the imaginary earth is a dilated event for him. Apparently these two identical events are not same for Observer O. According to Einstein an event occurred at any place is not simultaneous to two observers in a relative motion w.r.t to each other. Now the alternate definition of the same statement as per in the language of relativity is "two identical events in a relative motion are no longer simultaneous as viewed by a single observer". Similarly if we choose to view this fall from a moving system the trajectory as we will see is a hyperbola whereas classically the same trajectory is parabola. Now as Observer O calculates for the each event, he just finds that the value of g for the real earth is exactly 9.8 $m/s^2$, whereas that for the imaginary earth is 2.45m/s$^2$, indicating that the strength of the imaginary earth is going on decreasing. Both the postulates are inter-related. One predicts other verifies. The result of the 2nd postulate is completely consistent with the prediction made by the 1st postulate. In fact it is the 'time' that specifies the role of gravity. Although the mass of imaginary earth increases to twice its rest mass at this speed, but by very deep and careful examination one would come to the conclusion that this increase in mass does not matter. A question must arises here that with the increase in mass the gravitational field necessarily increases proportionally by the same rate. That is a wrong approach of one's mind. In fact mass increases, the gravitational field decreases i.e. the value of $G$ decreases. Accordingly increasing in mass in this case is absolutely meaningless. If someone tries to argue that with the increase in mass the gravitational field necessarily increases proportionally. A ridiculous idea which is logically incorrect but Let's accept for the time this ridiculous idea in order to check its validity. Consider another imaginary planet whose rest mass is half the rest mass of our native earth, but having the same volume i.e. the same radius as that of earth. Now in this case the value of 'g' for this imaginary planet is exactly the half as that for the real earth i.e. the value of 'g' for this imaginary earth is 9.8/2 = 4.9m /s$^2$. Now suppose that this imaginary earth is running with $v = 0.866c$ the real earth in the $+ve$ x-direction. With this speed its mass increases to twice its rest mass and thus equals to the rest mass of the real earth. Now as according to our supposition that with the increase in mass, the gravitational field necessarily increases proportionally by the same rate. Thus the value of g for this imaginary earth at this speed is 9.8 $m/s^2$ instead of 4.9 $m/s^2$. Now let us once again consider the experiment of free fall bodies. Let again two objects are released freely from the height of 490 meters at the same time above the each planet. Also consider that the object falling near the imaginary planet too is moving with $v = 0.866c$ in the direction of the imaginary earth (Fig.10).

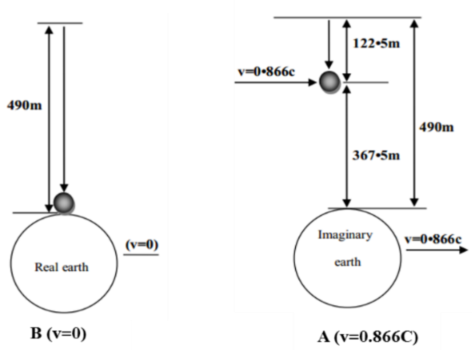


Fig 10

Both the bodies are at rest their respective planets. Now Let's examine this case. If someone claims that the strength of the field ultimately increases proportionally with the increase in mass. Then now at this stage the value of $g$ for this imaginary earth increases from 4.9m/s$^2$ to 9.8m/s$^2$ accordingly. Thus the observer resting on this imaginary earth i.e. Observer A registers that it takes exactly 10 seconds for the object to reach the imaginary planet. On the other hand the stationary observer i.e. Observer B measures the period of this fall for the same event to be 20 seconds. Now again the value of g in this case for the imaginary earth as measured by observer B is $g_B = 2.45\, m/s^2$, indicating that although the mass of imaginary earth is going on increasing, the gravitational field is going on decreasing. Now consider the same case in terms of two interacting bodies to see that how the gravitational force is affected at high speeds. Let two metallic balls of masses $m_1$ and $m_2$ each, are placed in a gravity free space and are separated by a small distance $r$ say ($r$ is an Arabic alphabet called Hamza ) of each other (Fig.11). Let both the balls are kept aloof of each other from gravitational pull by some external means say (i.e. by external forces say) as shown (Fig.11). Later on as the external forces are laid down, the gravitational attraction now will start to operate of each side. Let's find their relative velocity of approach attributable to gravitational pull as shown (Fig.12).

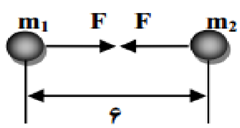


Fig 11

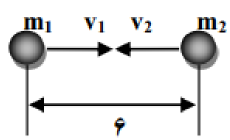


Fig 12

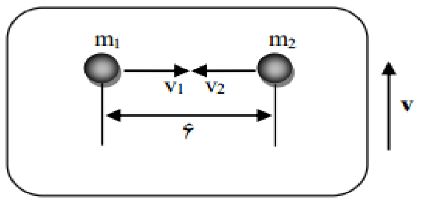


Fig 13

The gravitational force between the two bodies is $Gm_1m_2/(r)^2$ and the total available $K.E$ is equal to the total available $P.E$ i.e, $Gm_1m_2 / r$.

$$K.E_{Total} = P.E_{Total}$$

$$m_1 {v_1}^2 + m_2 {v_2}^2 = 2\,G m_1 m_2/\mathrm{r} \qquad (1)$$

Along with this the momentum must be conserved of each side.

$$i.e \quad m_1 v_1 = m_2 v_2 \qquad (2)$$

Now from Equ (1) we have.

$$m_1 {v_1}^2 + m_2 {v_2}^2 = 2\,G m_1 m_2/\mathrm{r}$$

$$v_1(v_1 + v_2) = 2\,G m_2/\mathrm{r} \qquad (3)$$

By similar mathematical treatment we have.

$$v_2(v_1 + v_2) = 2\,G m_1/\mathrm{r} \qquad (4)$$

Adding equations (3) & (4) we have.

$$v_1(v_1 + v_2) + v_2(v_1 + v_2) = 2\,G(m_1 + m_2)/\mathrm{r}$$

$${v_1}^2 + v_1.v_2 + v_1.v_2 + {v_2}^2 = 2\,G(m_1 + m_2)/\mathrm{r}$$

$${v_1}^2 + 2v_1.v_2 + {v_2}^2 = 2\,G(m_1 + m_2)/\mathrm{r}$$

$$(v_1 + v_2)^2 = 2\,G(m_1 + m_2)/\mathrm{r}$$

We have $(v_1 + v_2) = \sqrt{2\,G(m_1 + m_2)/r}$ but $(v_1+v_2) = v_{rel}$ the relative velocity of approach $v_{rel} = \sqrt{2\,G(m_1 + m_2)/\mathrm{r}}$. This is the relative velocity of approach. In case when $m_1 = m_2 = m$ say then $v_{rel} = 2\sqrt{Gm/r}$. The instantaneous change in this velocity is the gravitational acceleration i.e. $a_{grav}$ . Now

$$\frac{d}{dt}(v_{rel}) = a_{grav}$$

Upon integration yields

$$\int_0^{v_{rel}} dv_{rel} = \int_0^t a_{grav}.dt$$

$$v_{rel} = a_{grav}.t$$

We have

$$a_{grav} = v_{rel}/t = 2\sqrt{Gm/r}/\,t \qquad (5)$$

Now suppose that this entire mass is enclosed in evacuated box of negligible mass say and the entire system is moving relativistically with high speed in the upward direction (Fig.13). Now for an observer observing this whole situation from any rest frame in space will note that this particular time of approach slows down to him by an amount, $1/\sqrt{1 - v^2/c^2}$ i.e. $t = t'/\sqrt{1 - v^2/c^2}$ . Now Eq (5) takes the form as under.

$$a'_{grav} = 2\sqrt{Gm/r}\,/\,(t'/\sqrt{1 - v^2/c^2})$$

$$a'_{grav} = 2\sqrt{1 - v^2/c^2}.\sqrt{Gm/r}/\,t'$$

$$a'_{grav} = 2\sqrt{G(1 - v^2/c^2)m/r}\,/t' \qquad (6)$$

Comparing Equ (5) and (6) clearly $a'_{grav}$ is less than $a_{grav}$. Also look at this last equation (6), once again 'G' is losing its weight by an amount $(1 - v^2/c^2)$, indicating that gravitational activities die near the speed of light. Also note that the direction of the box is not in the direction of the line joining these two bodies gravitationally. Hence there is no worry of the Lorentz contracted distance for any observer. As we saw in the case of interacting bodies, how the strength of the gravitational field decreases by an amount $(1 - v^2/c^2)$. Let's now try for another effort for further confirmation of the same result. Let two observers *O(1)* and *O(2)* say (both are sisters) renders their services in order to check this phenomenon. *O(2)* is resting on an earth along with a pendulum oscillating with a period of 2 seconds say and the earth is supposed to be absolutely at rest in a gravity free region (Fig.14), while the other observer i.e. *O(1)* is watching the same situation from any rest frame outside the earth in a space.


Fig 14

Both the observers are agreed about the same period i.e. about the same value of g. Now all of a sudden the earth starts running with a speed $v = 0.866c$, then what will happen? Again time dilation will play its role. The period of oscillation is same for O(2) i.e. 2 seconds, but O(1) disagrees with O(2) and finds out the following period for oscillation as.

$$t_{(N)} = t_{(H)}/\sqrt{1 - v^2/c^2} = 4\,sec$$

The rest observer i.e. O(1) claims that the period is, $t_N = 4$ seconds, while for the moving observer i.e. for O(2) the period of the same event is, $t_H = 2$ seconds. Now what about the value of g for the each observer, Let's see.

| O(1) Point Of View | O(2) Point Of View |
|---|---|
| $v = 0, \quad t_{(N)} = 4$ seconds | $v = 0.866c, \quad t_{(H)} = 2$ seconds |
| $t_{(N)} = 2\pi\sqrt{l/g_{(N)}}$ | $t_{(H)} = 2\pi\sqrt{l/g_{(H)}}$ |
| $g_{(N)} = 4\pi^2 l/(t_N)^2$ | $g_{(H)} = 4\pi^2 l/(t_H)^2 = 4\pi^2 l/(2\,sec)^2$ |
| $g_{(N)} = 4\pi^2 l/(4\,sec)^2$ | $g_{(H)} = \pi^2 l$ |
| $g_{(N)} = \frac{1}{4}\pi^2 l$ | $=> \quad g_{(H)} = 4g_{(N)}$ |

Again both the observers does, t agree about the value of g. Strong disagreement exists between the observers about the value of 'g'. O(1) measures this value to be $g_N$, while that for O(2) the value is $g_H = 4g_N$, i.e. four times greater than that for O(1). A very unhappy situation. The principle of relativity has been badly demolished. Now a third observer Amna say is looking for this situation carefully. She quickly understands the problem. The whole of situation is clear to her. She too wants to render her services. She comes to O(2) and advised her that if she does not want to violate the principle of relativity then she will have to do a little modification i.e. she will have to lessen the value of the gravitational constant *G* by an amount $(1 - v^2/c^2)$ and thus she will find herself to be escaped of this troublesome situation. As she does so, she has the same value of 'g' as that for *O(1)*.

Let's see how *O(2)* handles this situation by decreasing the value of gravitational constant $G$ by an amount $(1 - v^2/c^2)$.

| For *O(1)* | For *O(2)* |
|---|---|
| $g_{(N)} = GM/R^2$ | $g_{(H)} = 4g_{(N)} = 4\,GM/R^2$<br>Now decreasing 'G' by an amount $(1 - v^2/c^2)$<br>$g_{(H)} = 4G(1 - v^2/c^2)M/R^2$<br>$g_{(H)} = 4G[1 - (0.866)^2]M/R^2$<br>$g_{(H)} = 4G(0.25)M/R^2 = GM/R^2$<br>$=> \quad g_{(H)} = g_{(N)}$ |

The value of g now is the same for both observers, i.e. the laws of physics are same for both. The day is an Eid-day for each one. The physics of relativity has been survived of death.

**EXPLANATION**

Now consider another pendulum with the same length i.e. with the same period ($t$ =*2sec*). If this pendulum is taken up to a height of $R$ from the earth surface i.e. a distance of $2R$ from the earth center (Fig.15).

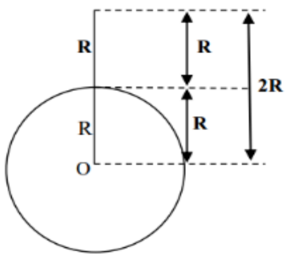


Fig 15

Now the period of the same pendulum definitely increases. Let's see that by how much amount the period of this pendulum increases. The value of g at the earth surface is, g = $GM/R^2$ and the period of this pendulum on the earth surface is, $t$ =*2sec*. At a height of $R$ from the earth surface, the total distance from the earth center is $2R$. Now Let's see what happens to the value of g a height of $2R$ above the earth center.

$$g' = GM/(2R)^2 = \frac{1}{4}[GM/R^2] = \frac{1}{4}g$$

At a height twice the radius of earth from the earth center the acceleration of gravity decreases to $(1/4)^{th}$ the value of g at the earth surface. The value of g at the surface of earth is four times greater than that at a height $R$ above the earth surface. Now Let's examine the period of each pendulum i.e. one at the earth surface, a distance of $R$ from the earth center and the $2^{nd}$ one at a height $R$ above the earth surface, i.e. at a distance of $2R$ from its center.

| $1^{st}$ Pendulum (Earth surface) | $2^{nd}$ Pendulum ( $h = 2R$) |
|---|---|
| $Length\ of\ the\ pendulum = l$<br>$Acceleration\ of\ gravity = g$<br>$t = 2\pi\sqrt{l/g} = 2\ seconds$ | $Length\ of\ the\ pendulum = l$<br>$g' = \frac{1}{4}g$<br>$t' = ?$<br>$Now,\quad t' = 2\pi\sqrt{l/g'} = 2\pi\sqrt{l/(\frac{1}{4}g)}$<br>$t' = 2\pi\sqrt{4\,l/g} = 2(2\pi\sqrt{l/g}\,)$<br>$t' = 2(2\ sec) = 4\ sec$ |

The pendulum at the earth surface has a period of 2 seconds, whereas that for a height $R$ above the earth surface is 4 seconds, despite the fact that both the pendulums are identical i.e. each has the same length. In other words while the $2^{nd}$ pendulum a height $R$ above the earth surface completes one oscillation, the earth based pendulum has completed two oscillations in the same time. Now Let's once again consider the case of two planets in a relative motion. One is our native earth call it the real earth, the $2^{nd}$ is an imaginary one. Both the planets are identical in all respect i.e. each the planet has the same rest mass and the same volume. The real earth is completely at rest in a gravity free region, while the imaginary one is running with $v = 0.866c$ in the $+ve$ x-direction to the real one (Fig.16).

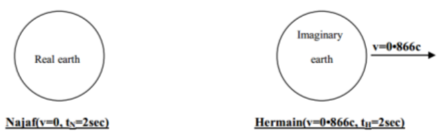


Fig 16

Now again the two observers O(1) and Herman play their role. O(1) is standing on the real earth which is at rest in a space, while O(2) is resting on the imaginary earth moving with $v = 0.866c$. Each the observer is equipped with an identical pendulum of period, $t = 2$ sec each. Each observer measures the same period of oscillation in their respective coordinate system as the principle of relativity requires. For O(1) the period of her own pendulum is $t_N$ = 2seconds. For Herman the period of her own pendulum too is $t_H$ = 2seconds. Now Let's analyse this whole situation with O(1),s point of view. For O(1) the period of her own pendulum is $t_N$ = 2seconds. As she turns her attention from her own frame and focuses on the imaginary one, she just finds out that Herman's pendulum is not oscillating with a period of 2seconds, but it takes a longer time to oscillate, i.e.

$$t_{(N)} = t_{(H)}/\sqrt{1 - v^2/c^2} = 4\ sec$$

The stationary observer i.e. O(1) finds that it takes 4 seconds to complete one oscillation and corresponding to this her own pendulum resting on the real earth has completed two oscillations in this time. Clearly O(1) concludes that the gravitational field of the imaginary earth has been decreased. As she calculates, she exactly finds out that the gravitational field of the imaginary earth has been decreased to $(1/4)^{th}$ of its rest field. Look carefully that this situation is completely analogous to the first one. In the first case the earth-based pendulum is completing two oscillations, while the 2nd pendulum at a distance of $2R$ from the earth center has completed only one oscillation in the same time. Now in the 2nd case the imaginary earth-based pendulum completes only one oscillation, while the real earth-based pendulum has completed two oscillations in the same time. This very close resemblance of these two situations clearly indicates that the gravitational field of the imaginary earth is going on decreasing. In the first case the field decreases due to increase in altitude, in the 2nd case the field decreases because of motion. Let's now examine for another aspect. In the year 1905 a young physicist of 26, Albert Einstein in his most popular writing (Bernstein, Fishbane, & Gasiorowicz, 2000) in which he proposed the theory of relativity, Einstein proposed a test. Einstein imagined two identical clocks and putting one of them at the north pole and the other at the equator (Fig.17).

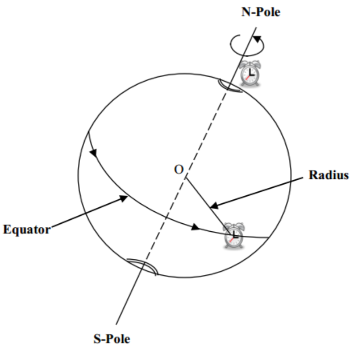


Fig 17

Now at any instant the motion of a point at the equator is just simply a motion in a straight line with a constant speed while the pole is at rest w.r.t the equator. The time dilation effect now will play its role. As a result the clock at the equator will run slowly w.r.t the polar clock. If the polar observer reads time $t_p$ with his clock, then he will register that the equatorial clock reads $t_e$. Now according to the polar observer

$$t_p = t_e / \sqrt{1-(v_{eq})^2/c^2} = t_e[1-(v_{eq})^2/c^2]^{-1/2}$$
$$i.e,\ t_p = t_e[1+(v_{eq})^2/2c^2]$$

here $v_{eq}$ is the velocity of a point at the equator. The fractional time shift between the polar clock and the equatorial clock is.

$$(t_p - t_e)/t_e = (v_{eq})^2/2c^2$$

This is the fractional time shift between both the clocks. Obviously the equatorial clock runs slowly w.r.t the polar clock by an amount $(v_{eq})^2/2c^2$. A decade later by bringing general relativity into practice Einstein noted that not only the motion but gravity too affects the measurement of time. For this purpose Einstein imagined an emitter say $E$ that emits radiation of frequency $f$ say a height 'h' above the earth surface and the detector $D$ is lying on the surface of earth (Fig.18).

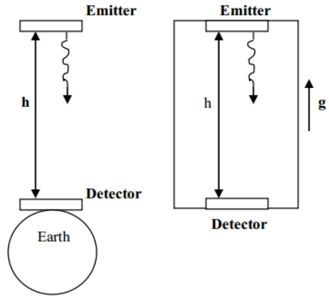


Fig 18

Both are in a vertical fashion. The emitter is at a height 'h' above the earth surface i.e. at a higher potential w.r.t the detector. So the gravitational potential difference between the emitter and the detector is $\Delta\varphi = gh$. Now suppose that this entire system is enclosed in the Einstein's elevator and the elevator is accelerated with acceleration g in the upward direction continuously(Fig.18). Now according to the principle of equivalence both the situations are same. Now as the elevator is accelerated upward with acceleration g , then during this acceleration a pulse of radiation is emitted at $t = 0$. Now in the time $t = h/c$ the radiation will reach the detector. During this time the detector will have acquired a speed $v = gt = gh/c$. This is the case of Doppler effect, in which the observer moves towards the emitter or source. Thus the detector sees that the frequency $f$ of radiation has increased as below.

$$f' = f(1 + v/c)$$

$$f'/f = 1 + v/c = 1 + (1/c)[gh/c] = 1 + gh/c^2$$

$$f'/f = 1 + \Delta\varphi/c^2$$

Now according to the principle of equivalence the frequency of light coming towards earth also will increase. Similarly the frequency of light rising away from earth will decrease. Now the fractional change in the frequency is,

$$(f' - f)/f = \Delta f/f = \Delta\varphi/c^2$$

Now Let's consider the solar system. We receive light from sun and the sun is too much massive than earth. Potential at the sun surface is $GMS/R_s$ ($M_S$ is the mass of and $R_s$ is the radius of the sun) and w.r.t sun we are at zero potential, so the change in potential approximately is all due to sun. Now the frequency of light reaching us is going on decreasing by an amount.

$$\Delta f/f = \Delta\varphi/c^2 = (1/c^2)[-GMs/Rs]$$
$$\Delta\varphi/c^2 = -GMs/Rs.c^2$$

If we look into the sign, the potential difference between the sun and earth is $-ve$, so the frequency of light reaching us decreases. Now we look at the other aspect. We know that the frequency of a periodic system is related to the period $T$ by $T = 1/f$ i.e. $f = 1/T$. Now differentiating $f = 1/T$ time we have.

$$\frac{df}{dt} = \frac{d}{dt}\left(\frac{1}{T}\right) = \frac{-1}{T^2}$$

$$(\Delta f/\Delta T)_{\Delta T\to 0} = -1/T^2$$

so we have $\Delta f = -\Delta T/T^2$. Thus

$$\Delta f/f = -\Delta T/T = \Delta\varphi/c^2$$

This is the fractional shift in the system period. Now the fractional shift in the total elapsed time is.

$$\Delta t/t = -\Delta\varphi/c^2 = GMs/Rs.c^2$$

Look again at the sign. The sign is $+ve$, the time increases. The clock at the earth surface will run quickly than that at the sun surface. An observer At any potential will see a clock higher in potential running more quickly and will see an identical clock in the lower potential running slowly. If we have two identical clocks keeping one at the earth surface (sea level) and the other at the top of the mountain K-2 at a higher potential. Then the clock at the K-2,s peak will run slightly a little faster than the earth-based clock. In this way Einstein came to the conclusion that not only motion but gravity too is responsible to affect the time. Now the combined effect due to the motion of equatorial clock w.r.t polar clock along with any difference $\Delta\varphi$ in the gravitational potential between the pole and the equator results in a net shift.

$$\Delta t/t = (v_{eq})^2/2c^2 - (-\Delta\varphi/c^2)$$

$$\Delta t/t = (v_{eq})^2/2c^2 + \Delta\varphi/c^2$$

Now although the polar and equatorial clocks both are acted upon by the same gravitational force towards the center of earth, but the equatorial clock is subject to an additional force i.e. the centripetal force. The equatorial clock is accelerated towards the earth center by centripetal acceleration due the axial rotation as, $a_{centrip} = (v_{eq})^2/R = \omega^2 R$ (here $\omega$ is termed as omega).

We know that a force can be expressed in terms of potential energy as a negative of the space derivative of potential energy. The associated acceleration (i.e force per unit mass) may be expressed as a negative of the space derivative of the potential (i.e. P.E per unit mass). i.e. centripetal acceleration = – *d/dr* (centripetal potential).

$$a_{centrip} = -\frac{d}{dr}[\varphi_{centrip}]$$

$$-\int d\varphi_{centrip} = \int a_{centrip}\,.dr$$

$$-\varphi_{centrip} = \int [(v_{eq})^2/r]dr = \int [\omega^2 r^2/r]\,dr = \omega^2\int r dr$$

$$= \omega^2 r^2/2 = (v_{eq})^2/2$$

$$\varphi_{centrip} = -(v_{eq})^2/2$$

For the equatorial clock, this potential must be included along with gravitational potential to get a net potential. i.e. Net potential (eq) = gravitational potential + centripetal potential

$$\varphi(net)_{eq} = \varphi(grav)_{eq} + \varphi_{centrip}$$

i.e. $\varphi(net)_{eq} = \varphi(grav)_{eq} - (v_{eq})^2/2$

For the polar clock there is no additional acceleration, so $\phi_{centrip(pol)} = 0$ for the polar clock. The only potential is the gravitational potential. Thus the net potential for the polar clock is. Net potential(pole) = gravitational potential(pole)

$$\varphi(net)_{pol} = \varphi(grav)_{pol}$$

Now according to Einstein the net potential at the equator must be equal to the net potential at the pole.

$$\varphi(net)_{eq} = \varphi(net)_{pol}$$

$$\varphi(grav)_{eq} - (v_{eq})^2/2 = \varphi(grav)_{pol}$$

$$\varphi(grav)_{eq} = \varphi(grav)_{pol} + (v_{eq})^2/2 \quad (7)$$

i.e. the gravitational potential at the equator is higher than that at the pole by an amount $v^2_{(eq)}/2$. This difference in potential between the pole and the equator ultimately will affect the time. Now Let's find the net fractional time shift due to equatorial motion together with a shift due to the gravitational potential.

$$\Delta t/t = (v_{eq})^2/2c^2 + \Delta\varphi/c^2$$

$$(t_p - t_e)/t_e = (v_{eq})^2/2c^2 + (\varphi_{pol} - \varphi_{eq})/c^2$$

$$(t_p - t_e)/t_e = (v_{eq})^2/2c^2 + [\varphi_{pol} - \{\varphi_{pol} + (v_{eq})^2/2\}]/c^2$$

From Eq (7)

$$\Delta t/t = (v_{eq})^2/2c^2 + \left[\varphi_{pol} - \varphi_{pol} - (v_{eq})^2/2\right]/c^2$$

$$= (v_{eq})^2/2c^2 - (v_{eq})^2/2c^2 = 0$$

There is no fractional shift at all between the two clocks. Both the clocks run at the same rate and completely synchronized. The effect on time produced by a potential difference between the pole and the equator effectively cancels that produced by the equatorial motion. Later on Einstein himself admitted too that he was wrong in 1905. Now Let's once again return to the case of real and the imaginary earth. The imaginary earth has the same rest mass as the real earth, the same volume i.e. the same radius and the same period of axial rotation, i.e. all the characteristics of the imaginary earth is same as that of real earth (Fig.19).

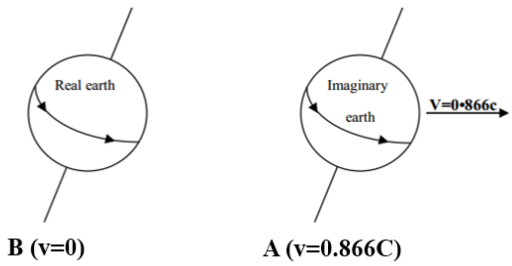


Fig 19

Now Let's suppose that the real earth is completely at rest in a gravity free region. The only kind of motion is its axial rotation and let the imaginary earth is moving with $v = 0.866c$ in a +ve x-direction w.r.t the real earth in a gravity free region with the same axial rotation (Fig.19). Let's again consider two observers Observer B and Observer A say. Observer B is resting on the real earth and Observer A is resting on the imaginary one. Each the observer is agree about the same period of axial rotation for their respective coordinate systems. According to Observer B the real earth completes one rotation in $t_J$ =24 hours, while Observer A also measures that the imaginary earth ($v = 0.866c$) too takes 24 hours to complete one axial rotation, i.e. $t_F$=24 hours. Each the observer claims too that the polar and equatorial clocks run at the same rate for their respective planets, as required by the relativity rule. Now Let's see the rotation of the imaginary earth from Observer B's point of view(i.e. the real earth observer). According to Observer A (i.e, the imaginary earth observer) his own planet i.e. the imaginary earth completes one axial rotation in $t_F=24$ hours. On the other hand Observer B (i.e. the real earth observer) when turns his attention and focused on the imaginary earth moving with $v = 0.866c$ him, he finds something like this.

$$t_{(B)} = t_{(A)}/\sqrt{1 - v^2/c^2} = 48 \text{ hour}$$

Obviously Observer B observes that it takes 48 hours (by his clock) for the imaginary earth to complete one rotation. According to Observer B the real earth has completed two rotations about its axis in this time, i.e. for observer B the real earth completes one rotation in 24 hours, while the imaginary earth takes 48 hours to complete one rotation. Now according to Observer B the angular velocity $\omega$ of the imaginary earth has decreased to one half of the real earth i.e. if the real earth completes $\pi$ radians then the imaginary earth completes only $\pi/2$ radians in the same time. Consequently the linear velocity $V$ of a particle at the equator of the imaginary earth also decreases to one half as that of the real one, according to the relation, $v = r\,\omega$. For the real earth the angular velocity = $\omega$ The linear velocity of a particle at the equator of real earth, $v = R\omega$. The angular velocity of the imaginary earth, $\omega' = \omega/2$. The linear velocity of a particle at the equator of imaginary earth, $v' = R\omega' = R\omega/2 = v/2$ $v_{img} = v_{real}/2$ Now the centripetal acceleration $a_{cent}$ of the imaginary earth as viewed by Observer B is

$$a_{cent(img)} = (v_{img})^2/R = (v_{real}/2)^2/R = \frac{1}{4}\,[(v_{real})^2/R]$$

$$a_{cent(img)} = \frac{1}{4}[a_{cent(real)}]$$

The centripetal acceleration of a point at the equator of the imaginary earth decreases to one fourth time that of the real earth as measured by Observer B. Now the associated centripetal potential $\varphi_{cent}$ of the imaginary earth of which this acceleration has been derived definitely will fall by the same rate. As a result the clock at the equator of this imaginary earth will run slowly in this lower potential. On the other hand the linear velocity $v'$ of a point at the equator of this imaginary earth decreases to one half that of the real earth. The time dilation effect produced by the equatorial motion will diminish. As a result the equatorial clock of the imaginary earth will start to run quickly due to this decrease in motion. The decrease in potential slows down the equatorial clock, while the decrease in motion results in speeding up the same clock. Both the factors (i.e. decrease in potential +decrease in motion) necessarily cancels the effect of each other and no net effect will be seen in the equatorial clock of the imaginary earth. Both the polar and equatorial clocks of the imaginary earth run at the same rate as viewed Observer B(i.e. the real earth observer). Now after having this discussion Let's return back to the case of centripetal acceleration. As we saw previously that the centripetal acceleration of the imaginary earth (v=0.866c) falls to one fourth (1/4$^{th}$) that of the real earth, $a_{cent(img)} = \frac{1}{4}[a_{cent(real)}]$ , the centripetal force associated with this acceleration must decreases by the same rate, i.e. $F_{cent(img)} = \frac{1}{4}[F_{cent(real)}]$. The centripetal force of the imaginary earth also decreases to one fourth (1/4)$^{th}$ that of the real earth. As we saw that the centripetal force of the imaginary earth is going on decreasing, now our prime objective is to see whether the gravitational force of the imaginary earth may decreases? In order to see that how the gravitational force decreases, we will have to see the behavior of force at high speed. Suppose that a force $F_o$ of constant magnitude acts on the Einstein's elevator moving upward with high speed and whose rest mass is $m_o$ say(Fig.20).

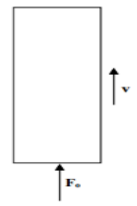

Fig 20

As the speed of an object cannot exceed the speed of light $C$, it is very hard to accelerate the elevator and so the Newton's 2$^{nd}$ law is no more applicable in this case and therefore must be modified properly. We know that $F_o = \frac{dP}{dt} = \frac{d}{dt}(m_o v)$ . But at high speeds the relativistic formulation of momentum is $P = m_o v/\sqrt{1 - v^2/c^2}$ . Now the 2$^{nd}$ law takes the form $F_o = \frac{d}{dt}(P) = \frac{d}{dt}(m_o v/\sqrt{1 - v^2/c^2})$. This is the relativistic formulation of Newton's 2$^{nd}$ law. Now

$$F_o = \frac{d}{dt}(P) = \frac{d}{dt}(m_o v/\sqrt{1 - v^2/c^2})$$

$$F_o = \left(m_o/\sqrt{1 - v^2/c^2}\right).\frac{dv}{dt} + v.\frac{d}{dt}(m_o/\sqrt{1 - v^2/c^2})$$

$$F_o = m_o v^\bullet/\sqrt{1 - v^2/c^2} + v.m_o\frac{d}{dt}(1 - v^2/c^2)^{-1/2}$$

$$F_o = m_o v^\bullet/\sqrt{1 - v^2/c^2} + m_o.v[\frac{-1}{2}(1 - v^2/c^2)^{-3/2}.\frac{d}{dt}(1 - v^2/c^2)]$$

$$F_o = m_o v^\bullet/\sqrt{1 - v^2/c^2} + m_o.v[\frac{-1}{2}(1 - v^2/c^2)^{-3/2}.(\frac{-1}{c^2}.2v.\frac{dv}{dt})]$$

$$F_o = m_o v^\bullet/\sqrt{1 - v^2/c^2} + m_o.v[(v.v^\bullet/c^2)(1 - v^2/c^2)^{-3/2}]$$

$$F_o = \frac{m_o v^\bullet}{\sqrt{1 - v^2/c^2}} + \frac{m_o v^\bullet(v^2/c^2)}{(1 - v^2/c^2)^{3/2}} = \frac{m_o v^\bullet}{\sqrt{1 - v^2/c^2}}(1 + \frac{v^2/c^2}{\sqrt{1 - v^2/c^2}})$$

$$F_o = \frac{m_o v^\bullet}{\sqrt{1 - v^2/c^2}}[\frac{1 - v^2/c^2 + v^2/c^2}{1 - v^2/c^2}]$$

$$F_o = \frac{m_o v^\bullet}{\sqrt{1 - v^2/c^2}}[\frac{1}{1 - v^2/c^2}]$$

But we know that $m_o/\sqrt{1 - v^2/c^2} = m$ and $v^\bullet = dv/dt = a$. So we have $F_o = ma/(1 - v^2/c^2)$. Hence $ma = F_o(1 - v^2/c^2)$ . So if a constant force acts on the Einstein's elevator say, moving upward with high speed, then what we would expect? Though in relativity a force does not produce a constant acceleration and indeed acceleration is the function of time elapsed. The instantaneous acceleration just produced in the elevator is small and a man standing on the weighing scale in the elevator, the scale will register a less reading. The downward force exerted by a man or any object placed on the weighing scale is reduced by an amount $(1-v^2/c^2)$. This outstanding result also holds good for the case of gravitational field. The principle of equivalence also requires to do so. In terms of gravitational field, $a =$ g and $F_o = F_{grav}$ then we have.

$$mg = F_{grav}(\,1 - v^2/c^2)$$

Clearly, the mass of an object or anybody else resting on the imaginary earth running with high speed is going on increasing, its weight is going on decreasing. The gravitational field of the imaginary earth has been decreased by an amount $(1-v^2/c^2)$. In case if the imaginary earth is moving with $v = 0.866c$, its strength will decrease by $[1-(0.866)^2\,] = 0.25 = 1/4$. Thus the gravitational force of an imaginary earth moving with v=0.866c also decreases to $(1/4)^{th}$ of its rest force as in the case of centripetal force.

## COLLABORATION

Let's see the beauty and eternity of this result on a broad level that how the measurement of gravity is affected by motion. This is our solar system consisting of a heavy star at the center and having nine planets revolving around, let including Pluto too obeying well-known Kepler's laws (KEPLER'S) and also some of the satellites revolving around their respective planets under the same Keplerian's picture. The working principle necessary for this entire system is the gravitational force. Now Let's suppose that at any later instant all the planets revolving around the sun suddenly stops their motion around the sun and similarly at the same instant all the satellites revolving around their respective planets also comes to rest, then what will happen? After freezing the entire solar system in this way all the planets along with their respective satellites will starts to fall one after one towards the sun. Eventually a time will reach that the entire solar system will coalesce into a single body. Let's now see that how this fascinating and fantastic event starts and comes to an end. Taking a start from the mercury first as it is the nearest neighbour to sun. Suppose that $M_s$ be the mass of sun and let $m$ be the mass of mercury. Let at any instant $t$ say during the fall $\alpha$ be the distance between their centers (Fig.21).

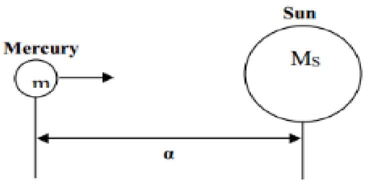


Fig 21

Now the gravitational force acting on mercury is.

$$F = -GM_s m/\alpha^2 \qquad (8)$$

We know that $F = ma = m\frac{dv}{dt} = m\frac{dv}{d\alpha}\cdot\frac{d\alpha}{dt}$ . Putting this expression in equation (8) we have.

$$mv\cdot\frac{dv}{d\alpha} = -GM_s m/\alpha^2 \quad \text{i.e. } v.dv = \frac{-GM_s}{\alpha^2}d\alpha$$

Integrating we get $\int vdv = -GM_s\int(\alpha)^{-2}d\alpha$

$$\frac{v^2}{2} = -GM_s\left[\frac{(\alpha)^{-1}}{-1}\right] + A$$

$$\frac{v^2}{2} = \frac{GM_s}{\alpha} + A \qquad (9)$$

Let $\beta$ be the distance between the mercury and sun at the instant when the mercury just refuse to revolve around the sun (Fig.22), then we have.

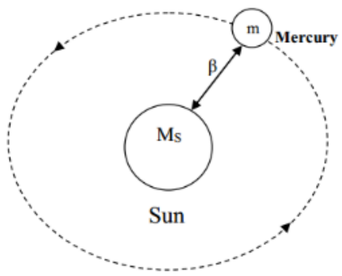


Fig 22

We know that $v = 0$, when $\alpha = \beta$. Therefore Eq(9) becomes as,

$$\frac{(0)^2}{2} = \frac{GM_s}{\beta} + A \quad \text{so} \quad A = \frac{-GM_s}{\beta}\ .$$

Now putting this in equation (9) we have.

$$\frac{v^2}{2} = \frac{GM_s}{\alpha} - \frac{GM_s}{\beta} = GM_s(\frac{1}{\alpha} - \frac{1}{\beta})$$

$$v^2 = 2GM_s\left[\frac{\beta-\alpha}{\alpha.\beta}\right] = \frac{2GM_s}{\beta}\left[\frac{\beta-\alpha}{\alpha}\right]$$

Taking the square root we have. $v = \sqrt{\frac{2GM_s}{\beta}}\cdot\sqrt{\frac{\beta-\alpha}{\alpha}}$

Now $v = \frac{d\alpha}{dt} = \sqrt{\frac{2GM_s}{\beta}}\cdot\sqrt{\frac{\beta-\alpha}{\alpha}}$ then,

$$dt = \sqrt{\frac{\beta}{2GM_s}}\cdot\frac{\sqrt{\alpha}}{\sqrt{\beta-\alpha}}\cdot d\alpha$$

Integrating when $t = 0, v = 0$ and $\alpha = \beta$. So we have

$$\int_0^t dt = \sqrt{\frac{\beta}{2GM_s}}\cdot\int_\beta^0\frac{\sqrt{\alpha}}{\sqrt{\beta-\alpha}}\cdot d\alpha$$

$$t = \sqrt{\frac{\beta}{2GM_s}}\cdot\int_\beta^0\frac{\sqrt{\alpha}}{\sqrt{\beta-\alpha}}\cdot d\alpha \qquad (10)$$

Let $\sqrt{\beta-\alpha} = y$ , then $\beta-\alpha = y^2$ and $\beta - y^2 = \alpha$ . Now $\frac{d}{d\alpha}(\beta-\alpha) = \frac{d}{d\alpha}(y^2)$ , we have $0-\frac{d\alpha}{d\alpha} = 2y.\frac{dy}{d\alpha}$ $-d\alpha = 2ydy$ , thus we have $d\alpha = -2ydy$. When α = β, then y=0, when α = 0, then y =$\sqrt{\beta}$ . Now equation (10) takes the form.

$$t = \sqrt{\frac{\beta}{2GM_s}}\cdot\int_0^{\sqrt{\beta}}\frac{\sqrt{\beta-y^2}}{y}\cdot(-2ydy)$$

$$t = -2.\sqrt{\frac{\beta}{2GM_s}}\cdot\int_0^{\sqrt{\beta}}\sqrt{\beta-y^2}\cdot dy \qquad (11)$$

$$t = -2.\sqrt{\frac{\beta}{2GM_s}}\cdot\int_0^{\sqrt{\beta}}\sqrt{\left(\sqrt{\beta}\,\right)^2-y^2}\cdot dy$$

if $y = \sqrt{\beta}$ then,

$\sqrt{\beta-y^2} = \sqrt{\left(\sqrt{\beta}\,\right)^2-y^2} = \sqrt{\left(\sqrt{\beta}\,\right)^2-\left(\sqrt{\beta}\,.\sin f\right)^2} = \sqrt{\beta(1-\sin^2 f)} = \sqrt{\beta}.\sqrt{1-\sin^2 f} = \sqrt{\beta}.\sqrt{\cos^2 f} = \sqrt{\beta}.\cos f$

i.e. $\int \sqrt{\beta - y^2} \cdot dy = \int \sqrt{\beta}.\cos f . \sqrt{\beta}.\cos f df = \beta \int \cos^2 f df = \frac{\beta}{2}\int 2\cos^2 f df$

$$= \frac{\beta}{2}\int (1+\cos 2f)\, df$$

$$\int \sqrt{\beta - y^2}\cdot dy = \frac{\beta}{2}\int df + \frac{\beta}{2}\int \cos 2f\, df = \frac{\beta}{2}(f) + \frac{\beta}{2}\left[\frac{\sin 2f}{2}\right] + k$$

where k, is constant of integration

$$\int \sqrt{\beta - y^2}\cdot dy = \frac{\beta}{2}(f) + \frac{\beta}{2}\left[\frac{2\sin f\cos f}{2}\right] + k$$

$$\int \sqrt{\beta - y^2}\cdot dy = \frac{\beta}{2}(f) + \frac{\beta}{2}[\sin f \cos f] + k \quad (12)$$

But $\sqrt{\beta}.\sin f = y$, we have $f = \sin^{-1}(\frac{y}{\sqrt{\beta}})$, and $\frac{\beta}{2}(\sin f) = \frac{\sqrt{\beta}.\sqrt{\beta}.\sin f}{2} = \frac{y.\sqrt{\beta}}{2}$, and $\cos f = \sqrt{1-\sin^2 f} = \sqrt{1-\frac{y^2}{\beta}}$, now with these substitutions Eq (12) becomes as,

$$\int \sqrt{\beta - y^2}\cdot dy = \frac{\beta}{2}\cdot \sin^{-1}(\frac{y}{\sqrt{\beta}}) + \frac{y.\sqrt{\beta}}{2}\cdot\sqrt{1-\frac{y^2}{\beta}} + k$$

$$\int \sqrt{\beta - y^2}\cdot dy = \frac{\beta}{2}\cdot \sin^{-1}(\frac{y}{\sqrt{\beta}}) + \frac{y}{2}\sqrt{\beta - y^2} + k \quad (13)$$

Now substituting equation(13) in equation(11) we have,

$$t = -2.\sqrt{\frac{\beta}{2GM_s}}\cdot\left[\frac{\beta}{2}\cdot\sin^{-1}\left(\frac{y}{\sqrt{\beta}}\right) + \frac{y}{2}\sqrt{\beta - y^2}\right]_0^{\sqrt{\beta}} + k$$

Now applying the initial conditions i.e. when $t = 0$, $\alpha = \beta$ then $y = 0$, so we have, $k = 0$

i.e. $t = -2.\sqrt{\frac{\beta}{2GM_s}}\cdot\left[\frac{\beta}{2}\cdot\sin^{-1}\left(\frac{\sqrt{\beta}}{\sqrt{\beta}}\right) + \frac{\sqrt{\beta}}{2}\sqrt{\beta-\beta}\right]$

$$t = -2.\sqrt{\frac{\beta}{2GM_s}}\cdot\left[\frac{\beta}{2}\cdot\sin^{-1}1 + 0\right]$$

$$= -2.\sqrt{\frac{\beta}{2GM_s}}\cdot\left[\frac{\beta}{2}\cdot\sin^{-1}1\right]$$

$$= \frac{-\sin^{-1}1.\beta^{3/2}}{\sqrt{2GM_s}}$$

After simplification we dropping the –ve sign, we have the exact time for fall as.

$$t = \frac{90\,\beta^{3/2}}{\sqrt{2GM_s}} \quad (14)$$

Here $M_S$ is the mass of sun and $\beta$ is the distance between the mercury and sun. This is the time required for mercury to embrace with sun. All the planets gradually will fall to sun number-wise one by one under the parachute of the same equation. Eventually the entire solar system will coalesce into a single body. The last victim of the sun's gravity is Pluto. Now Let's consider another solar system (i.e. an imaginary solar system) having the same size, the same rest mass as our real solar system and similarly all the planets of the imaginary system are alike that of the real solar system, briefly another facsimile of the real solar system. Now suppose that this entire solar system is running uniformly with a high speed in the $+ve$ x-direction w.r.t the real one absolutely at rest in the space. Let the imaginary solar system too is undergoing the same process. For an observer having appointment on the imaginary sun, the equations of physics and mathematics are same as that for the real solar system. This observer resting on the imaginary sun will find that the bombardment of planets toward the imaginary sun follow exactly the same equation as that for the real solar system. For an observer resting on the imaginary sun the time for fall of any planet to the imaginary sun is.

$$t_{img} = \frac{90\,\beta^{3/2}}{\sqrt{2GM_s}}$$

Now the question arise, whether the observer observing this entire process from the vantage place of the rest frame of the real solar system is agree with the observer resting on the imaginary sun for the same time for the fall of planets to the imaginary sun? No, the real sun's observer will find that it takes a longer time for fall than that for the imaginary sun's observer. For the real sun's observer the time for this fall is.

$$t_{real} = t_{img} \times \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{90.\beta^{3/2}}{\sqrt{2GM_s}} \times \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{90.\beta^{3/2}}{\sqrt{2G(1 - v^2/c^2)M_s}}$$

Open your mind please and look carefully, what is happening with $G$ ? Again $G$ is losing its weight by an amount (1- $v^2/c^2$). The gravitational field of the entire solar system is going on decreasing by the same rate. Now for an instance we suppose that if the real solar system takes 100 years to coalesce into a single body, then the observer resting on the imaginary sun exactly will measure that it too requires 100 years, for the imaginary solar system to coalesce into a single body. But on the other hand the real sun's observer will register that it takes $100/\sqrt{1 - v^2/c^2}$ years for this imaginary solar system to well-mix. For example if this entire solar system is running with $v = 0.6c$, the imaginary sun's observer claims that it takes 100 years for coalescence. But on the other hand the real sun's observer will find something like this much.

$$t_{real} = t_{img} \times \frac{1}{\sqrt{1 - v^2/c^2}} = 125\ years$$

Obviously for the real sun's observer 125 years are required for integration into a single body. Similarly for example, if $v_{img} = 0.866c$, then again $t_{img} = 100 years$ and $t_{real} = \frac{100}{\sqrt{1-(0.866)^2}} = \frac{100}{0.5} = 200\ years$, i.e. if the imaginary solar system is running with $v = 0.866c$, then again time required for integration according to the imaginary observer is 100 years and that for the real sun's observer the time of integration is 200 years for the same event.

In case of strong field less amount of time is required to coalesce into a single body, for a weak field definitely longer time is required for coalescence. In case if $v_{img} = 0.866c$, the value of 'G' for the imaginary system as measured from the rest frame of the real solar system is, $G(1 - v^2/c^2) = G[-(0.866)^2] = \frac{1}{4}G$ the gravitational field of the imaginary system decreases to one fourth (1/4)th of its rest field as observed from the rest frame of the real solar system.

## EXPERIMENTAL PROOF

(The precession of the perihelion of mercury)

Consider a single solar system (Kenneth, 1983) consisting of a massive sun of mass $M_s$ and a single light planet revolving around elliptically according to the Kepler's first law of the planetary motion having the sun as one focus of the ellipse. According to the Newtonian gravitation (inverse square law) the orbit is a perfect ellipse with the sun at one focus. The equation for the ellipse is.

$$r = r_{min}.\frac{1+e}{1+e.\cos\theta}$$

Here $r_{min}$ is the minimum distance between a planet ang the sun and $e$ is the eccentricity of the ellipse. By eccentricity we mean the degree to which an ellipse is non-circular. For ellipse $e$ is always less than unity, for circle $e=0$, for parabola $e=1$ and that for parabola $e$ is always greater than unity, whereas $r = r_{min}$ is the closest approach of the planets to the sun and at $r = r_{min}$ the planet is said to be at its perihelion. This occurs regularly exactly at the same point in space, where $\theta = 0$, $2\pi, 4\pi, 6\pi$, $8\pi$… etc. Now for the case of mercury the orbit followed by mercury is not quite a closed ellipse.

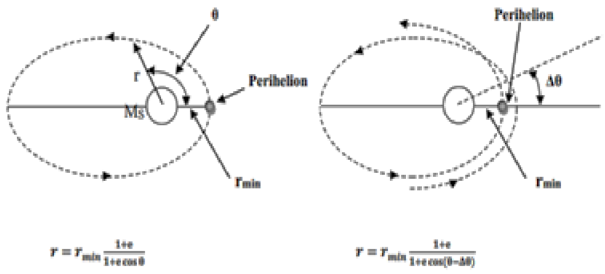


Fig 23

As the mercury passes its perihelion, the closest point of approach to the sun (Fig.23) the perihelion slightly little shifts of its original path by an amount $\Delta\theta$ say. After completing one orbit the mercury returns to $r_{min}$ but a slightly different θ, i.e. $\Delta\theta$ say. Now in such a case we have.

$$r = r_{min}.\frac{1+e}{1+e\cos(\theta-\Delta\theta)}$$

Although this shift $\Delta\theta$ is an extremely a very small quantity, but this effect is cumulative i.e. it builds up orbit after orbit and after $N$ orbits say, the perihelion has advanced by an amount $N\Delta\theta$. As a result over many orbits, the perihelion will slowly rotate about the sun. This effect is termed as 'the precession of the perihelion'. Now the question arises, why this precession occurs ? Consider a particle of mass $m$ in a circular motion having a linear momentum $\overline{p}$ at a position vector $\vec{r}$ relative to the centre $O$ fixed in an inertial frame as shown (Fig.24).

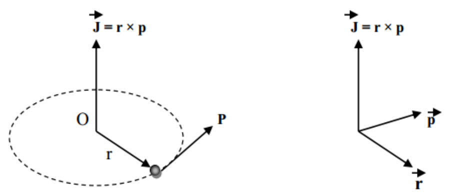


Fig 24

The angular momentum of a particle about $O$ is a vector represented by symbol '$\vec{J}$' and is defined as $\vec{J} = \vec{r} \times \vec{p}$. The direction of '$\vec{J}$' is perpendicular to the plane formed by $\vec{r}$ and $\vec{p}$ according to the right hand rule and its magnitude is equal to the area of the plane formed by $\vec{r} \times \vec{p}$.

According to the law of conservation of angular momentum "in the absence of any external torque the angular momentum is constant" i.e. $\vec{J} = \vec{r} \times \vec{p} = mvr = m\omega r^2$ = constant. Now according to the Kepler's 2nd law for planetary motion (Russell, 1964) "the areal velocity is constant in the planetary motion i.e. the line joining the centers of a planet and the sun sweeps out equal areas in equal intervals of time". This is called the law of areas. The Kepler's 2nd law is in agreement with the law of conservation of angular momentum. According to the Kepler's 1st law (Russell, 1964) all the planets execute their motion in elliptical orbits around the sun, having the sun as one focus of the ellipse. Now in order to conserve the angular momentum each planet has to move faster at the point of the closest approach to the sun than at the farthest point. This is so because at these points the angular momentum is $mvr$ and this has to be conserved all the way. Thus when $r$ becomes shorter $V$ must becomes larger in order to keep the angular momentum equal to $mvr$ (Fig.25).

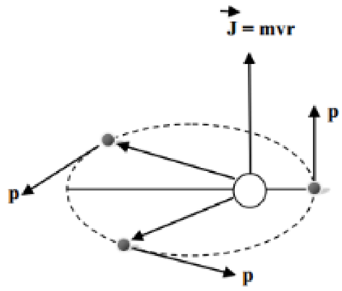


Fig 25

Now look at this point carefully. As the distance between the mercury and sun decreases, its velocity increases. With this increase in velocity the gravitational field of mercury gradually decreases. At the perihelion the mercury has the highest speed and its field has reached to its lowest level. This weak field is no more able to cope with the angular momentum vector $\vec{J}$. The victorious angular momentum vector $\vec{J}$ now is overpowering the economy of the mercury's weak field at this stage. As a result the perihelion of the mercury shifts little of its right course. Although the speed of mercury is non-relativistic and no doubt that this change in the mercury's field is extremely a very small, but in fact this small change in the mercury's field is responsible for a small portion of this precession along with the combined effect due to general relativity. In fact it is the dominant role of $\vec{J}$ (angular momentum vector) responsible for the fractional part of the complete precession. Although Kepler's laws showed that the planetary motion could be described with a great simplicity if the sun was taken as a reference body, but their reality and physical significance were not fully appreciated to the rest of the world. These laws were purely empirical in nature i.e. based on observations and experiments only without any theoretical interpretation. In formulating these laws Kepler had no concept of force as a main cause of the observed regularities. Thanks to Isaac Newton, the genius for all times by born that took a bold step in this respect and successfully extended the concept of force to the Kepler's laws and was then able to formulate the law of universal gravitation. Newton's gravitational law for the planetary motion demanded each planet to be attracted towards the sun with an "inverse square force". In this way Newton successfully accounted for the motion of the planets in the solar system and the bodies falling near the earth surface with one common concept. A great triumph indeed, no doubt. No doubt, Newton by his very deep and profound insight effectively accounted for what was responsible for the mechanism of falling bodies near the earth surface as well as for keeping the planets in orbital motion around the sun and so many contributions to science often tribute to this very great genius man of the history. But despite all these achievements and full time success, he had no idea of the decreasing fields. That is why the inverse square law fails to stand with this peculiar effect. Not only the general relativity alone, but special relativity too is responsible for this effect up to some extent.

It is now a time to have a respite for a while and to release the tension of much discussion and lengthy calculations. Let's consider this interesting situation before going into more detail. Let two fast persons, Sapna and Sohana, having equal rest masses. Sapna is resting on the real earth at rest in a space whereas her other friend Sohana is resting on the imaginary earth identical to the real one in all respect, moving with $v = 0.866c$ relative to the real one. Let each the friend became a pregnant at the same time having fair dealings. Both the friends are standing on a weighing scale. The rest observer i.e. Sapna has a lovely child in her lap exactly after nine months. Let's see this situation with Sohana's point of view i.e. the imaginary earth observer. According to Sohana, Sapna,s mass is less than me while her weight is greater than me according to the reading of the scale. Also the period of Sapna's pregnancy as appeared to Sohana is $t_{Sohana} = 9\sqrt{1-(0.866)^2} = 4.5\, months$. According to Sohana her friend has delivered the child before the maturity time. Now Let's see the same situation with Sapna,s point of view. According to Sapna, Sohana,s mass is twice her rest mass but her weight is three fourth $(3/4)^{th}$ times less than her rest weight as per scale reading. Also according to Sapna her friend is still pregnant after the due date. May be a serious medical problem with her friend. Both the friends does not know this dilemma. Very bad and a puzzling situation for each one but an interesting one for us.

## THE LAST ADVENTURE

The full and final attack will now be assaulted with the weapons of quantum physics, that firmly will establish the belief that G is dying. We have a natural unit of time called the Plank time denoted by $t_p$ that brings the relativity constant $c$, the gravitational constant 'G' and the quantum constant 'h' into a single picture and also tells the story of the times when the universe was in its infancy. The Plank time $t_p$ the shortest unit of time ever formulated which on the dimensional grounds has the form. Plank time,

$$t_p = \sqrt{\frac{G.\hbar}{c^5}}$$

The dilated shape of the Plank time $t_p$ is,

$$t_p = \frac{t'_p}{\sqrt{1-v^2/c^2}}$$

or

$$t'_p = t_p\sqrt{1-v^2/c^2}\,.$$

Now we have

$$t'_p = (\sqrt{1-v^2/c^2}).\sqrt{\frac{G.\hbar}{c^5}} = \sqrt{(1-v^2/c^2)\cdot\frac{G.\hbar}{c^5}}$$

Now here arises three possibilities with the entrance of the relativistic factor $1/\sqrt{1-v^2/c^2}$ into Plank time $t_p$ .

**First possibility:** Will the entrance of the relativistic factor i.e. $1/\sqrt{1-v^2/c^2}$ into Plank time change $c$, the speed of light in a free space? This is impossible for $c$ to be affected. The speed of light $c$ is same for all observers when measured and where measured irrespective of their state of motion, completely independent of any relative motion**.**

**Second possibility:** This possibility arises for $h$, the Plank constant. Will $h$ be affected with the entrance of this relativistic factor into Plank time? We know that $h$ is Plank constant that comes in Plank energy equation i.e. $E = h\nu$. With any change in Plank constant $h$, consequently the entire pattern of the energy conservation rules ultimately will change. However the relativists of the entire world have their firm belief that the laws of conservation of momentum as well as that of energy both holds good in the world of relativity. Thus $h$ too is preventing itself of any change.

**Third possibility:** The unfortunate lottery now falls into the lap of $G$. Ultimately the advent of the relativistic factor into Plank time is going to change $G$ by an amount $(1-v^2/c^2)$. Now the complete dilated shape of the Plank time $t_p$ is.

$$t'_p = \sqrt{\frac{G(1-v^2/c^2).\hbar}{c^5}}$$

Thus the Plank time $t_p$ is dilated by an amount, $1/\sqrt{1-v^2/c^2}$ at the cost of $G$. Another approach to do the same can be made easy with the help of Plank length $l_p$.

## PLANK LENGTH

$$l_p = \sqrt{G.\hbar/c^3}\ ,$$

Now the contracted shape of the Plank length i.e. $l'_p$ is

$$l'_p = l_p.\sqrt{1-v^2/c^2} = \sqrt{G(1-v^2/c^2).\hbar/c^3}$$

Contraction occurs in length at the cost of $G$. Now Let's consider the case of Plank mass i.e. $m_p = \sqrt{\hbar.c/G}$ . The relativistic version of the Plank mass i.e. $m'_p$ is

$$m'_p = \frac{m_p}{\sqrt{1-v^2/c^2}} = \frac{\sqrt{\hbar.c/G}}{\sqrt{1-v^2/c^2}} = \sqrt{\frac{\hbar.c}{G(1-v^2/c^2)}}$$

i.e. mass is going on increasing at the cost of 'G'. Let's consider now the case of Plank charge $q_p$. Plank charge, $q_p = \sqrt{4\pi\varepsilon_o\hbar c}$ Here we see that $G$ plays no role in this case. Now what can we conclude? The answer is that the electric charge 'q' is an invariant quantity and cannot be affected by any relative motion and has the same value in all frames. Now we take into account the case of Plank temperature $T_p$.

## PLANK TEMPERATURE

$$T_p = \sqrt{\hbar . c^5 / G . {K_B}^2}$$

Here $K_B$ that appears in this expression is Boltzmann constant i.e. the universal gas constant per molecule. Now look at, if $G$ changes its character in the same way as in the previous cases, then what will happen? Look at the Plank temperature carefully. Clearly the internal temperature of the system increases at high speeds at the cost of $G$.

## SUMMARY OF THE OUTCOMES & COROLLARY

Our common judgment about the gravity is something like this much, that the gravitational constant *G* is completely a constant character, completely independent of any relative motion. Now but by all accounts *G* is proving itself to be completely a relative character and that *G* is not independent of motion. A new theoretical discipline, hitherto an unseen, unheard para in the stories of relativity. Gravity and time are inter-related to each other. Gravity alters time, time alters gravity as well. The space geometry now is $(x, y, z, t, G)$.

As a test, we have the universal gravitational constant *G* is absolutely a relative phenomenon. In order to check this phenomenon practically, Let's propose an idealized test. If Mercury and Pluto were of same mass and if we place two identical masses, one on Pluto and the other on mercury along with the instruments of accurate and precise measuring the weights and suppose that if we have the mechanism of switching off the sun's gravity. Then in the absence of sun gravity, Kepler's laws will fly-off and each of the planet will move in a straight line with mercury running faster than Pluto. Now the observers on each planet will agree that the value of *G* is less for mercury than that for Pluto. If mercury is running with velocity *V* relative to Pluto then.

$$G_{Mercury} = G_{Pluto}(1 - v^2/c^2)$$

If our earth suddenly starts running with the speed of light say (contradiction to special relativity) then what will happen? Yes all the inhabitants of earth will find that the gravitational field of earth has been switched-off and Einstein's space curvature will fly-off. The distant galaxies receding away from us with high velocities in the vast universe are enjoying the facility of the low cost of *G*.

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